

## Abstract

Over a  $p$ -adic local field  $F$  of characteristic zero, we develop a new type of harmonic analysis on an extended symplectic group  $G = \mathbb{G}_m \times \mathrm{Sp}_{2n}$ . It is associated to the Langlands  $\gamma$ -functions attached to any irreducible admissible representations  $\chi \otimes \pi$  of  $G(F)$  and the standard representation  $\rho$  of the dual group  $G^\vee(\mathbb{C})$ , and confirms a series of the conjectures in the local theory of the Braverman-Kazhdan proposal (Braverman and Kazhdan, 2000) for the case under consideration. Meanwhile, we develop a new type of harmonic analysis on  $\mathrm{GL}_1(F)$ , which is associated to a  $\gamma$ -function  $\beta_\psi(\chi_s)$  (a product of  $n + 1$  certain abelian  $\gamma$ -functions). Our work on  $\mathrm{GL}_1(F)$  plays an indispensable role in the development of our work on  $G(F)$ . These two types of harmonic analyses both specialize to the well-known local theory developed in Tate's thesis (Tate, 1950) when  $n = 0$ . The approach is to use the compactification of  $\mathrm{Sp}_{2n}$  in the Grassmannian variety of  $\mathrm{Sp}_{4n}$ , with which we are able to utilize the well developed local theory of Piatetski-Shapiro and Rallis (1986) and many other works) on the doubling local zeta integrals for the standard  $L$ -functions of  $\mathrm{Sp}_{2n}$ .

The method can be viewed as an extension of the work of Godement-Jacquet (1972) for the standard  $L$ -function of  $\mathrm{GL}_n$  and is expected to work for all classical groups. We will consider the Archimedean local theory and the global theory in our future work.