

# Contents

Chapter 1. Introduction	1
1.1. Main results	5
1.2. Key ideas and further results	7
1.3. Open problems	11
1.4. Organization and notation	13
Acknowledgements	14
Chapter 2. Preliminaries	15
2.1. Continued fractions	15
2.2. Ostrowski expansions	17
2.3. Bohr sets	19
2.4. Measure theory	21
2.5. Real analysis	21
2.6. Geometry of numbers	23
2.7. Primes and sieves	25
Chapter 3. A fully inhomogeneous version of Gallagher's theorem	27
3.1. Notation and reduction steps	27
3.2. Divergence of the series	29
3.3. Overlap estimates, localized Bohr sets, and the small-GCD regime	35
3.4. Large GCDs	40
3.5. A convergence statement	45
Chapter 4. Liouville fibres	49
4.1. A special case	49
4.2. Diophantine second shift	51
4.3. Liouville second shift	55
4.4. Rational second shift	59
Chapter 5. Obstructions on Liouville fibres	61
Appendix A. Pathology	67
Bibliography	71