Abstract

We define higher quantum Airy structures as generalizations of the Kontsevich–Soibelman quantum Airy structures by allowing differential operators of arbitrary order (instead of only quadratic). We construct many classes of examples of higher quantum Airy structures as modules of $\mathcal{W}(\mathfrak{g})$ algebras at self-dual level, with $\mathfrak{g} = \mathfrak{gl}_{N+1}$, $\mathfrak{so}_{2N}$ or $\mathfrak{e}_N$. We discuss their enumerative geometric meaning in the context of (open and closed) intersection theory of the moduli space of curves and its variants. Some of these $\mathcal{W}$ constraints have already appeared in the literature, but we find many new ones. For $\mathfrak{gl}_{N+1}$ our result hinges on the description of previously unnoticed Lie subalgebras of the algebra of modes. As a consequence, we obtain a simple characterization of the spectral curves (with arbitrary ramification) for which the Bouchard–Eynard topological recursion gives symmetric $\omega_{g,n}$s and is thus well defined. For all such cases, we show that the topological recursion is equivalent to $\mathcal{W}(\mathfrak{gl})$ constraints realized as higher quantum Airy structures, and obtain a Givental-like decomposition for the corresponding partition functions.