Abstract

Motivated by the limit mixed Hodge structure on the Milnor fiber of a hypersurface singularity germ, we construct a natural mixed Hodge structure on the torsion part of the Alexander modules of a smooth connected complex algebraic variety. More precisely, let U be a smooth connected complex algebraic variety and let $f\colon U \to \mathbb{C}^*$ be an algebraic map inducing an epimorphism in fundamental groups. The pullback of the universal cover of $\mathbb{C}^*$ by f gives rise to an infinite cyclic cover U^f of U. The action of the deck group $\mathbb{Z}$ on U^f induces a $\mathbb{Q}[t^{\pm 1}]$-module structure on $H_*(U^f;\mathbb{Q})$. We show that the torsion parts $A_*(U^f;\mathbb{Q})$ of the Alexander modules $H_*(U^f;\mathbb{Q})$ carry canonical $\mathbb{Q}$-mixed Hodge structures. We also prove that the covering map $U^f \to U$ induces a mixed Hodge structure morphism on the torsion parts of the Alexander modules. As applications, we investigate the semisimplicity of $A_*(U^f;\mathbb{Q})$, as well as possible weights of the constructed mixed Hodge structures. Finally, in the case when $f\colon U \to \mathbb{C}^*$ is proper, we prove the semisimplicity and purity of $A_*(U^f;\mathbb{Q})$, and we compare our mixed Hodge structure on $A_*(U^f;\mathbb{Q})$ with the limit mixed Hodge structure on the generic fiber of f.