Abstract

We consider the one-dimensional nonlinear Schrödinger equation with a nonlinearity of degree $p > 1$. On compact manifolds many probability measures are invariant by the flow of the *linear* Schrödinger equation (e.g. Wiener measures), and it is sometimes possible to modify them suitably and get *invariant* (Gibbs measures) or *quasi-invariant* measures for the non linear problem. On $\mathbb{R}^d$, the large time dispersion shows that the only invariant measure is the δ measure on the trivial solution $u = 0$, and the good notion to track is whether the non linear evolution of the initial measure is well described by the linear (nontrivial) evolution. This is precisely what we achieve in this work. We exhibit measures on the space of initial data for which we describe the nontrivial evolution by the linear Schrödinger flow and we show that their nonlinear evolution is absolutely continuous with respect to this linear evolution. Actually, we give precise (and optimal) bounds on the Radon–Nikodym derivatives of these measures with respect to each other and we characterise their L^p regularity. We deduce from this precise description the global well-posedness of the equation for $p > 1$ and scattering for $p > 3$ (actually even for $1 < p \leq 3$, we get a dispersive property of the solutions and exhibit an almost sure polynomial decay in time of their L^{p+1} norm). To the best of our knowledge, it is the first occurence where the description of quasi-invariant measures allows to get quantitative asymptotics (here scattering properties or decay) for the nonlinear evolution.