Abstract

We consider refined conjectures of Birch and Swinnerton-Dyer type for the Hasse–Weil–Artin L-series of abelian varieties over general number fields. We shall, in particular, formulate several new such conjectures and establish their precise relation to previous conjectures, including to the relevant special case of the equivariant Tamagawa number conjecture. We also derive a wide range of concrete interpretations and explicit consequences of these conjectures that, in general, involve a thoroughgoing mixture of difficult Archimedean considerations related to refinements of the conjecture of Deligne and Gross and delicate p-adic congruence relations that involve the bi-extension height pairing of Mazur and Tate and are related to key aspects of noncommutative Iwasawa theory. In important special cases we provide strong evidence, both theoretical and numerical, in support of the conjectures.