Abstract

We introduce a theory of stratifications of noncommutative stacks (i.e., presentable stable ∞-categories), and we prove a reconstruction theorem that expresses them in terms of their strata and gluing data. This reconstruction theorem is compatible with symmetric monoidal structures, and with more general operadic structures such as $\mathbb{E}_n$-monoidal structures. We also provide a suite of fundamental operations for constructing new stratifications from old ones: restriction, pullback, quotient, pushforward, and refinement. Moreover, we establish a dual form of reconstruction; this is closely related to Verdier duality and reflection functors, and gives a categorification of Möbius inversion.

Our main application is to equivariant stable homotopy theory: for any compact Lie group G, we give a symmetric monoidal stratification of genuine G-spectra. In the case that G is finite, this expresses genuine G-spectra in terms of their geometric fixedpoints (as homotopy-equivariant spectra) and gluing data therebetween (which are given by proper Tate constructions).

We also prove an adelic reconstruction theorem; this applies not just to ordinary schemes but in the more general context of tensor-triangular geometry, where we obtain a symmetric monoidal stratification over the Balmer spectrum. We discuss the particular example of chromatic homotopy theory.