

## Abstract

We continue the study of multiple cluster structures in the rings of regular functions on  $GL_n$ ,  $SL_n$  and  $\text{Mat}_n$  that are compatible with Poisson–Lie and Poisson-homogeneous structures. According to our initial conjecture, each class in the Belavin–Drinfeld classification of Poisson–Lie structures on a semisimple complex group  $\mathcal{G}$  corresponds to a cluster structure in  $\mathcal{O}(\mathcal{G})$ . Here we prove this conjecture for a large subset of Belavin–Drinfeld (BD) data of  $A_n$  type, which includes all the previously known examples. Namely, we subdivide all possible  $A_n$  type BD data into oriented and non-oriented kinds. We further single out BD data satisfying a certain combinatorial condition that we call aperiodicity and prove that for any oriented BD data of this kind there exists a regular cluster structure compatible with the corresponding Poisson–Lie bracket. In fact, we extend the aperiodicity condition to pairs of oriented BD data and prove a more general result that establishes an existence of a regular cluster structure on  $SL_n$  compatible with a Poisson bracket homogeneous with respect to the right and left action of two copies of  $SL_n$  equipped with two different Poisson–Lie brackets. Similar results hold for aperiodic non-oriented BD data, but the analysis of the corresponding regular cluster structure is more involved and not given here. If the aperiodicity condition is not satisfied, a compatible cluster structure has to be replaced with a generalized cluster structure. We will address these situations in future publications.