

Abstract

In a recent paper of Benson and Symonds, a new invariant was introduced for modular representations of a finite group. An interpretation was given as a spectral radius with respect to a Banach algebra completion of the representation ring. Our purpose here is to take these notions further, and investigate the structure of the resulting Banach algebras. Some of the material in that paper is repeated here in greater generality, and for clarity of exposition.

We give an axiomatic definition of an abstract representation ring, and representation ideal. The completion is then a commutative Banach algebra, and the techniques of Gelfand from the 1940s are applied in order to study the space of algebra homomorphisms to \mathbb{C} . One surprising consequence of this investigation is that the Jacobson radical and the nil radical of a (complexified) representation ring always coincide.

These notes are intended for representation theorists. So background material on commutative Banach algebras is given in detail, whereas representation theoretic background is more condensed.