Contents

Chapter 1. Introduction 1
1.1. The main result 4
1.2. Necessary conditions 6
1.3. An application to maximal functions 7
1.4. Fourier multipliers 9
1.5. Application to weighted norm inequalities 11
1.6. Organization and notation 11
Acknowledgments 13

Chapter 2. Necessary Conditions 15
2.1. The local integrability hypothesis 16
2.2. The reflexivity hypothesis 19

Chapter 3. Single scale sparse domination 23
3.1. A single scale estimate 23
3.2. A resolution of the identity 24
3.3. Single scale regularity 24

Chapter 4. Proof of the main result 29
4.1. A modified version of sparse forms 29
4.2. Proof of the main theorem given the inductive claim 30
4.3. Proof of the corollary of the main theorem 31
4.4. The inductive step 32

Chapter 5. Maximal operators, square functions and long variations 43
5.1. Maximal functions and ℓ^r-variants 43
5.2. Variation norms 45
5.3. Truncations of sums 46
5.4. Some simplifications for maximal operators 48

Chapter 6. Fourier multipliers 51
6.1. The main multiplier theorem 51
6.2. A result involving localizations of Fourier multipliers 53
6.3. Proof of the main multiplier theorem 54

Chapter 7. Sample applications 59
7.1. Operators generated by compactly supported distributions 59
7.1.1. Maximal functions 59
7.1.2. Variational operators 61

7.1.3. Lacunary maximal functions for convolutions associated with the wave equation 63
7.2. General classes of multipliers 64
7.2.1. Miyachi classes and subdyadic Hörmander conditions 66
7.2.2. Multiscale variants of oscillatory multipliers 68
7.3. Prototypical versions of singular Radon transforms 71
7.3.1. An approach via Fourier multipliers 72
7.4. Densities on spheres: Maximal singular integrals 73
7.5. On radial Fourier multipliers 76
7.6. Stein's square function 78
7.6.1. The case $1 < p \leq 2$ 78
7.6.2. The case $2 < p < \infty$ 80

Appendix A. Facts about sparse domination 81
A.1. Replacing simple functions 81
A.2. The Hardy–Littlewood maximal function 82
A.3. Operators associated with dilates of Schwartz functions 83

Appendix B. Sparse domination: Cases where $p = 1$ or $q = \infty$ 85
B.1. The case $p = 1, q < \infty$ 85
B.2. The case $p > 1$, $q = \infty$ 87
B.3. The case $p = 1$ and $q = \infty$ 92

Appendix C. Facts about fourier multipliers 95
C.1. Multiplication by smooth symbols 96
C.2. Independence of ϕ, Ψ in the finiteness of $\mathcal{B}[m]$ 96

Bibliography 99