Abstract

We introduce coordinates on the spaces of framed and decorated representations of the fundamental group of a surface with nonempty boundary into the symplectic group $\mathrm{Sp}(2n, \mathbf{R})$. These coordinates provide a noncommutative generalization of the parametrizations of the spaces of representations into $\mathrm{SL}(2, \mathbf{R})$ or $\mathrm{PSL}(2, \mathbf{R})$ given by Thurston, Penner, Kashaev and Fock–Goncharov. On the space of decorated symplectic representations the coordinates give a geometric realization of the noncommutative cluster-like structures introduced by Berenstein–Retakh. The locus of positive coordinates maps to the space of framed maximal representations. We use this to determine an explicit homeomorphism between the space of framed maximal representations and a quotient by the group $\mathrm{O}(n)$. This allows us to describe the homotopy type and, when $n = 2$, to give an exact description of the singularities. Along the way, we establish a complete classification of pairs of nondegenerate quadratic forms.