Abstract

This Memoir concerns fundamental rogue-wave solutions of the focusing nonlinear Schrödinger equation in the limit that the order of the rogue wave is large and the independent variables (x, t) are proportional to the order (the far-field limit). We first formulate a Riemann-Hilbert representation of these solutions that allows the order to vary continuously rather than by integer increments. The intermediate solutions in this continuous family include also soliton solutions for zero boundary conditions spectrally encoded by a single complex-conjugate pair of poles of arbitrary order, as well as other solutions having nonzero boundary conditions matching those of the rogue waves albeit with far slower decay as $x \to \pm\infty$. The large-order far-field asymptotic behavior of the solution depends on which of three disjoint regions $\mathcal{C}$ (the "channels"), $\mathcal{S}$ (the "shelves"), and $\mathcal{E}$ (the "exterior domain") contains the rescaled variables. On the region $\mathcal{C}$, the amplitude is small and the solution is highly oscillatory, while on the region $\mathcal{S}$, the solution is approximated by a modulated plane wave with a highly oscillatory correction term. The asymptotic behavior on these two domains is the same for all continuous orders. Assuming that the order belongs to the discrete sequence characteristic of rogue-wave solutions, the asymptotic behavior of the solution on the region $\mathcal{E}$ resembles that on $\mathcal{S}$ *but without the oscillatory correction term.* Solutions of other continuous orders behave quite differently on $\mathcal{E}$.