

## Abstract

In this work we introduce and study a new notion of amenability for actions of locally compact groups on  $C^*$ -algebras. Our definition extends the definition of amenability for actions of discrete groups due to Claire Anantharaman-Delaroche. We show that our definition has several characterizations and permanence properties analogous to those known in the discrete case. For example, for actions on commutative  $C^*$ -algebras, we show that our notion of amenability is equivalent to measurewise amenability. Combined with a recent result of Alex Bearden and Jason Crann, this also settles a long standing open problem about the equivalence of topological amenability and measurewise amenability for a second countable  $G$ -space  $X$ .

We use our new notion of amenability to study when the maximal and reduced crossed products agree. One of our main results generalizes a theorem of Matsumura: we show that for an action of an exact locally compact group  $G$  on a locally compact space  $X$  the full and reduced crossed products  $C_0(X) \rtimes_{\max} G$  and  $C_0(X) \rtimes_{\text{red}} G$  coincide if and only if the action of  $G$  on  $X$  is amenable. We also show that the analogue of this theorem does not hold for actions on noncommutative  $C^*$ -algebras.

Finally, we study amenability as it relates to more detailed structure in the case of  $C^*$ -algebras that fibre over an appropriate  $G$ -space  $X$ , and the interaction of amenability with various regularity properties such as nuclearity, exactness, and the (L)LP, and the equivariant versions of injectivity and the WEP.