

Contents

Chapter 1. Introduction	1
1.1. Amenable actions	1
1.2. The weak containment problem	5
1.3. Regular $X \rtimes G$ -algebras and type I C^* -algebras	6
1.4. Regularity properties	7
1.5. Outline of the paper	8
1.6. Conventions and notation	9
1.7. Acknowledgements	12
Chapter 2. The G -equivariant enveloping von Neumann algebra	13
2.1. The enveloping von Neumann algebra of a C^* -action	13
2.2. Central covers of covariant representations	17
2.3. The predual of the enveloping G -von Neumann algebra	18
Chapter 3. Amenable actions	21
3.1. Amenable actions of locally compact groups	21
3.2. Permanence properties of amenability	33
3.3. The equivalence of amenability and measurewise amenability	34
Chapter 4. The quasi-central approximation property	37
4.1. The weak quasi-central approximation property (wQAP)	37
4.2. Consequences for Fell bundles	42
Chapter 5. The weak containment property and commutant amenability	45
5.1. Injective representations and commutant amenability	46
5.2. Applications of the Haagerup standard form	52
5.3. Weak containment does not imply amenability	55
Chapter 6. Actions on $X \rtimes G$ -algebras and type I C^* -algebras	65
6.1. Amenability of regular $X \rtimes G$ -algebras	65
6.2. Actions on type I C^* -algebras with Hausdorff spectrum	69
Chapter 7. Regularity properties	73
7.1. Properties passing to the crossed product by an amenable action	73
7.2. Characterizing exactness via actions on G -injective algebras	77
7.3. The continuous G -WEP	78
Chapter 8. Some new developments and questions	81
8.1. A summary of new developments	81
8.2. Some questions	82

Bibliography

85