# Abstract

We study Hamiltonicity in random subgraphs of the hypercube $\mathcal{Q}^n$. Our first main theorem is an optimal hitting time result. Consider the random process which includes the edges of $\mathcal{Q}^n$ according to a uniformly chosen random ordering. Then, with high probability, as soon as the graph produced by this process has minimum degree $2k$, it contains $k$ edge-disjoint Hamilton cycles, for any fixed $k \in \mathbb{N}$. Secondly, we obtain a perturbation result: if $H \subseteq \mathcal{Q}^n$ satisfies $\delta(H) \geq \alpha n$ with $\alpha > 0$ fixed and we consider a random binomial subgraph $\mathcal{Q}^n_p$ of $\mathcal{Q}^n$ with $p \in (0,1]$ fixed, then with high probability $H \cup \mathcal{Q}^n_p$ contains $k$ edge-disjoint Hamilton cycles, for any fixed $k \in \mathbb{N}$. In particular, both results resolve a long standing conjecture, posed e.g. by Bollobás, that the threshold probability for Hamiltonicity in the random binomial subgraph of the hypercube equals $1/2$. Our techniques also show that, with high probability, for all fixed $p \in (0,1]$ the graph $\mathcal{Q}^n_p$ contains an almost spanning cycle. Our methods involve branching processes, the Rödl nibble, and absorption.