## Contents

Chapte	r 1. Introduction	1
Chapte	r 2. Prelude: An example	7
2.1.	Extremal Kähler metrics on surfaces	7
2.2.	Cartan's realization problem and Lie algebroids	8
2.3.	,	9
2.4.	The Lie algebroid of extremal Kähler surfaces	10
2.5.	Surface Metrics with $ \nabla K  = 1$	10
Chapte	r 3. G-structure groupoids and connections	13
3.1.	Principal bundles and $G$ -structures over orbifolds	13
3.2.	G-principal groupoids	14
	Connections on $G$ -principal groupoids	18
	G-structure groupoids	20
3.5.	G-structure groupoids with connection	21
	r 4. G-structure algebroids and connections	23
	G-principal algebroids	23
4.2.	1 1 0	26
4.3.	G-structure algebroids	28
4.4.	G-structure algebroids with connection	29
4.5.	9	30
4.6.	G-realizations of a G-structure algebroid with connection	32
Chapte	r 5. Construction of solutions	35
5.1.	The dictionary I	35
5.2.	The dictionary II	36
5.3.	Existence of solutions I	37
5.4.	Universality of solutions	38
5.5.	An elementary example: Metrics of constant curvature	39
Chapte	r 6. G-integrability	41
6.1.	The canonical $G$ -integration	41
6.2.	Existence of solutions II	45
6.3.	Obstructions to $G$ -integrability	47
6.4.	Computing the obstructions to $G$ -integrability	50
Chapte	r 7. Solutions to Cartan's Realization Problem	53
7.1.	Local solutions	53
7.2.	Complete solutions and complete $G$ -Realizations	57
7.3.	Strongly complete realizations	59

iv CONTENTS

Chapter 8. Symmetries and moduli space of solutions	63
8.1. Moduli space of germs of solutions	63
8.2. Moduli space of 1-connected solutions	65
8.3. Comparing the moduli spaces $\mathfrak{C}(A)$ and $\mathfrak{G}(A)$	66
Chapter 9. G-structure algebroids and geometric structures	71
9.1. Metric type	72
9.2. Symplectic type	75
9.3. Complex type	76
Chapter 10. Extremal Kähler metrics on surfaces	79
10.1. Existence of local solutions	79
10.2. Complete extremal Kähler metrics	84
10.3. Explicit solutions	86
10.4. Symmetries and moduli space of extremal Kähler metrics	88
Bibliography	91