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Preface

The reader may be a little confused at the words "Real Analysis" in the title of the book. Indeed, real analysis, like harmonic analysis, has a lot of contents nowadays, and the author cannot definitively state what are the subjects that the real analysis deals with.

I suppose that real analysis meant, at first, the field which corresponded to complex analysis and aimed to analyse and synthesize sets and functions on the real axis or the plane. However, complex variable methods have been deeply incorporated into the Fourier analysis which has been one of the great backbones of real analysis. Also the problems, the methods, and the connections of real analysis to other fields have changed drastically. As a consequence, the objects dealt with by real analysis have been diversified and have not always been limited to sets or functions on the Euclidean plane.

Real analysis is based on the real numbers, and it is naturally involved with practical mathematics. On the other hand, it has taken on subject matter from set theory, harmonic analysis, integration theory, probability theory, complex analysis, the theory of partial differential equations, and so on, and has provided these theories with important ideas and basic concepts. Such relations continue even today.

This book, with these backgrounds as its setting, contains the basic matter of "real analysis".

It is natural, in view of the object of this book, to introduce the definition of the real numbers, which is done by just mentioning characteristic properties.

The statements about sets and plane topology will be kept to an irreducible minimum.

Measurability of sets is one of the fundamental notions of Lebesgue integration theory. To define measurability, we apply the Carathéodory condition, although this method is not always intuitive. One of the merits of the choice is that it enables us to unify the measures on the plane and on general sets. Moreover, the Carathéodory condition is used to simplify the proof of the theorem on representation of linear functionals by measures.

The existence of Haar measure and integration on topological groups are not covered. For them, the reader may refer to the bibliography in the back of the book.

The basic theory of distributions and Fourier analysis is covered in chapters 7 and 8.

The last chapter is allotted to an introduction to wavelet theory. The theory of wavelets was born at the beginning of the 1980's from a practical purpose and, coincidentaly, a purely mathematical motive. At present, the lucid theory is constructed by an effective application of Fourier analysis.

Wavelet theory is also important in Fourier analysis. Roughly speaking, it enables us to treat a function and its Fourier transform more easily at the same time. Wavelet theory is now a useful tool of real analysis, like distribution theory.

I would like to thank Professors Hitoshi Arai and Kazuya Tachizawa for useful remarks and making graphs.

I thank all the staff of Iwanami Shoten. Espcially I would like to thank Mr. Hideo Arai, Ms. Mamiko Hamamo and Mr. Uichi Yoshida for generous help in preparing the manuscript.

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Preface to the English Edition

The frst mathematical textbook which I read was "Measure Theory", by P. R. Halmos.

Even now I remember clearly the polished exposition, so neatly put in order.

I have benefited, since then, from many books published in Eurpopean languages, and I have had the opportunity to make deep friendships with excellent mathematicians. Especially, I have spent a delightful time with the professors and the students of the University of Wisconsin in Madison.

Integration theory, distribution theory, and Fourier analysis are still basic subjects for all students who study analysis. Also, the theory of wavelets is becoming a new basic. I hope that this book will give students good training in these basics.

I thank Professor Katsumi Nomizu, who gave useful advice in translation. I am full of gratitude to the kindness of Ralph Sizer, who read through the manuscript and gave me polite and pertinent comments, and correction of English.

Finally, I thank the American Mathematical Society for giving me the opportunity to publish this book in English, and the staff there, especially Christine M. Thivierge and Vickie Ancona, for their kind assistance.

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