Preface to Part 1

The author's idea was that this textbook should be aimed at students of mathematics having a background in general university courses in probability theory and mathematical statistics. The textbook was written based on the courses given by the author for students of mathematical departments at Volgograd University, Volgograd, Russia, and Donetsk University, Donetsk, Ukraine.

Among the books which may be used as a first reading in mathematical statistics, we mention the books by Cramér [**9**] and van der Waerden [**34**], which have already become the cornerstones in statistics. These books are still an authority, and many generations of experts have been brought up with these books. Elements of mathematical statistics are an essential ingredient of other general courses on probability theory. Let us mention the textbooks by Gnedenko [**12**], Gikhman, Skorokhod, and Yadrenko [**11**], Rozanov [**27**], Sevast'yanov [**29**], Tutubalin [**33**], and Shiryaev [**30**]. The textbooks by Shmetterer [**31**], Ivchenko and Medvedev [**14**], and Kozlov and Prokhorov [**19**] can be regarded as thoroughly developed introductions into mathematical statistics. The books on mathematical statistics by Borovkov [**5**], [**6**] take a special rank among textbooks for undergraduate and postgraduate students.

In writing this book, the author has used Russian and foreign literature on mathematical statistics, as well as the experience and traditions of teaching probability at Volgograd University and Donetsk University. Let us mention here the books by Rao [**26**], Cox and Hinkley [**8**], van der Waerden [**34**], and Bickel and Doksum [**4**] that thoroughly work out, each in its own way, problems for teaching mathematical statistics.

Part 1 of this book begins with a presentation of sampling using one-dimensional samples (Chapter 1) and multidimensional samples (Chapter 2) as an example. The basic sample characteristics are introduced and their asymptotic and nonasymptotic properties are studied. Main distributions related to the multidimensional Gaussian distribution are defined.

Chapter 3 deals with the estimation of parameters of distributions. In this chapter, measures of quality of statistical estimators are introduced and some optimality criteria are given. Optimal estimation of a scale parameter and a location parameter is studied. For regular families of distributions, approaches leading to effective estimators based on the Cramér–Rao inequality are given.

Chapter 4 deals with the theory of sufficient statistics and its applications to the construction of optimal estimators of unknown parameters and parametric functions.

In Chapter 5, general methods for constructing statistical estimators of parameters of distributions are considered and the main properties of the corresponding estimators are established.

The limited size of the book did not allow us to include some important statistical procedures or to consider other topics in the theory of parametric estimation. Part 2 of the textbook will deal with problems related to testing statistical hypotheses. The author hopes that this textbook will enable the reader to work independently, using other sources, on the topics we only touch upon here. We would recommend the books by Wilks [**35**] and Lehmann [**21**] and the three-volume monograph by Kendall and Stuart [**16**]–[**18**]. Our textbook can be used in preparation for general courses on mathematical statistics as well as specialized courses on the subject.

The list of references at the end of Part 1 includes only references available for students in Russia and Ukraine and is by no means complete.

In the textbook, we use the common notational conventions: P and P_θ for probabilities; E and E_θ for mathematical expectations; D and D_θ for variances, etc. We use triple notation for theorems, lemmas, formulas, etc. Therefore, for example, Theorem 4.1.2 refers to Theorem 2 in Section 1 of Chapter 4. Sections are enumerated by double numbers: Section 1.4 stands for Section 4 in Chapter 1. The sign □ marks the end of a proof.

Preface to Part 2

Part 1 of this book dealt with the estimation of unknown parameters, while Part 2 is devoted to testing statistical hypotheses.

The theory of hypotheses testing appears, in more or less detail, in practically any textbook or monograph on mathematical statistics. We mention here the books by Lehmann [**34**], and Hájek and Šidák [**22**] that are entirely devoted to statistical tests, as well as the book by Borovkov and Mogul'skiĭ [**10**] that is devoted to asymptotic problems in testing statistical hypotheses.

Part 2 begins with an exposition of a general theory of testing (Chapter 1), that is, of problems related to testing statistical hypotheses in the scheme of general statistical experiments according to Ibragimov and Khas'minskiĭ [**25**], Barra [**2**], and Soler [**49**]. First, in Section 1.1, we deal with testing two hypotheses, we study the structure of the set formed by type I and type II error probabilities, and we introduce Neyman-Pearson tests, Bayes tests, and minimax tests. In Section 1.2, the theory of testing a finite number of simple hypotheses is presented and the most powerful tests, Bayes tests, and minimax tests are introduced. Section 1.3 deals with testing composite hypotheses and discusses different approaches to the definition of optimal tests. A relationship between tests and confidence intervals is investigated.

Chapter 2 deals with problems for asymptotically distinguishable families of simple statistical hypotheses in the scheme of general statistical experiments following the books [**47**] and [**37**]. A complete group of types of families of statistical hypotheses that can be asymptotically distinguished is introduced and characterization theorems are given, which enables one to determine the type to which a family of hypotheses belongs (Section 2.2). Complete asymptotic testing under the strong law of large numbers (Section 2.3) or under weak convergence (Section 2.4) of the logarithm of the likelihood ratio are presented. Section 2.5 deals with testing contiguous families of hypotheses.

Chapter 3 is devoted to goodness-of-fit tests for independent observations. The Kolmogorov test (Section 3.1), the Pearson test (Section 3.2), and the Smirnov test (Section 3.3) are considered in detail. Section 3.4 focuses on some other well-known goodness-of-fit tests.

Chapter 4 presents elements of sequential analysis applied to the problem of testing statistical hypotheses. Section 4.1 deals with the Bayes theory of sequential testing of, generally speaking, composite hypotheses. Sections 4.2 and 4.3 are devoted to the Wald sequential test for testing two simple hypotheses. Section 4.2 presents the basic properties of the Wald test and Section 4.3 establishes that the Wald test is optimal.

The list of references at the end of the book contains only those references that are directly related to the topics we treat in the book and is by no means a complete list of references on testing statistical hypotheses.

In Part 2 we follow the same system of notational conventions as in Part 1. We also enumerate theorems, lemmas, formulas, etc., in the same way as we did in Part 1.