Preface

Inverse problems of spectral analysis deal with the reconstruction of operators in a specified form, given certain spectral characteristics of the operators. Interest in such problems was initially inspired by quantum mechanics. The main inverse spectral problems have already been solved for Schrödinger operators and their finite-difference analogues, the Jacobi matrices (see V. A. Ambartsumian [**11**], G. Borg [**12**], N. Levinson [**14**], V. A. Marchenko [**1**], M. G. Krein [**6**], I. M. Gelfand and B. M. Levitan [**3**], B. M. Levitan and M. G. Gasymov [**7**], L. D. Faddeev [**10**], R. Newton [**16**], N. I. Akhiezer [**2**], A. R. Its and V. B. Matveev [**5**], V. E. Zakharov and A. B. Shabat [**4**], P. Deift and X. Zhou [**13**], V. A. Yurko [**17**], Yu. I. Lyubarskii and V. A. Marchenko [**15**], etc.). On the other hand, little is known about inverse spectral problems for wider classes of operators, such as arbitrary Hermitian matrices.

The present monograph focuses on inverse problems in the theory of small oscillations of systems with finitely many degrees of freedom. Given data obtained from observations of these oscillations, to solve an inverse problem means to find the potential energy of the system in question. Since the oscillations are small, the potential energy is given by a positive definite quadratic form, whose matrix is called the matrix of potential energy. Hence, the problem is to find a matrix belonging to the quite wide set of all positive definite matrices. This is a principal difference between the inverse problems studied in this monograph and inverse problems for discrete analogues of the Schrödinger operators, where only tridiagonal Hermitian matrices are considered.

Without loss of generality it can be assumed that the systems consist of finitely many material points (particles) $\alpha, \beta, \ldots$ of masses $m_\alpha, m_\beta, \ldots$, interacting with each other and with an external field. It is assumed that only a small portion of the particles is available for observation. The aim is to use observations obtained from this subset of particles to compute the reduced matrix of potential energy (**L**-matrix of the system), whose elements $\mathbf{L}(\alpha, \beta)$ are expressed in terms of elements $\mathbf{U}(\alpha, \beta)$ of the matrix of potential energy via the formula $\mathbf{L}(\alpha, \beta) = \mathbf{U}(\alpha, \beta)(m_\alpha m_\beta)^{-1/2}$, and to compute the masses of the particles, if possible.

The main results obtained in the monograph are the following:

- necessary and sufficient conditions are found for a portion of particles with observed oscillations to enable computation of the **L**-matrix of the entire system;
- conditions are found for extracting the required information on oscillations of an observable part of the system from its oscillations in a neighborhood of infinity;
- conditions are found for computing the **L**-matrix of the entire system by using the spectra of oscillations of the system and some of its perturbations.

For the reader's convenience, Chapters 1–7 of the monograph contain a detailed presentation of well-known results on inverse spectral problems for tridiagonal matrices, i.e., Jacobi matrices; see also [**18**, **19**]. The subsequent Chapters 8–17 contain proofs of necessary and sufficient conditions for a subset of observable particles to determine uniquely the system **L**-matrix and give a method for its computation. Here a complete description is provided of the class of matrices that can be found from observable data on q particles, and some model examples are given. In particular, in the case $q = 1$ it is possible to find the tridiagonal matrix only. The problem of computation of the particle masses is also considered. Chapter 18 presents a solution of the inverse problem of reconstructing a Hermitian matrix given its spectrum and the spectra of some of its perturbations. Chapter 19 deals with the inverse problem of multichannel scattering. The final six chapters (Chapters 20–25) describe numerical methods for solving the inverse problem of the theory of small oscillations. To make the exposition clear to a wide range of readers, the material relating to numerical modeling of solutions is presented in detail. This content also includes topics which we believe may be of interest to experts in numerical analysis. Specific examples are given to illustrate the features of the methods described.

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