Introduction

The Clay Mathematics Institute (CMI) grew out of the longstanding belief of its founder, Mr. Landon T. Clay, in the value of mathematical knowledge and its centrality to human progress, culture, and intellectual life. Discussions over some years with Professor Arthur Jaffe helped shape Mr. Clay's ideas of how the advancement of mathematics could best be supported. These discussions resulted in the incorporation of the Institute on September 25, 1998, under Professor Jaffe's leadership. The primary objectives and purposes of the Clay Mathematics Institute are "to increase and disseminate mathematical knowledge; to educate mathematicians and other scientists about new discoveries in the field of mathematics; to encourage gifted students to pursue mathematical careers; and to recognize extraordinary achievements and advances in mathematical research." CMI seeks to "further the beauty, power and universality of mathematical thinking."

Very early on, the Institute, led by its founding scientific board — Alain Connes, Arthur Jaffe, Edward Witten, and Andrew Wiles — decided to establish a small set of prize problems. The aim was not to define new challenges, as Hilbert had done a century earlier when he announced his list of twenty-three problems at the International Congress of Mathematicians in Paris in the summer of 1900. Rather, it was to record some of the most difficult issues with which mathematicians were struggling at the turn of the second millennium; to recognize achievement in mathematics of historical dimension; to elevate in the consciousness of the general public the fact that, in mathematics, the frontier is still open and abounds in important unsolved problems; and to emphasize the importance of working toward solutions of the deepest, most difficult problems.

After consulting with leading members of the mathematical community, a final list of seven problems was agreed upon: the Birch and Swinnerton-Dyer Conjecture, the Hodge Conjecture, the Existence and Uniqueness Problem for the Navier–Stokes Equations, the Poincaré Conjecture, the **P** versus **NP** problem, the Riemann Hypothesis, and the Mass Gap problem for Quantum Yang–Mills Theory. A set of rules was established, and a prize fund of US\$7 million was set up, this sum to be allocated in equal parts to the seven problems. No time limit exists for their solution.

The prize was announced at a meeting on May 24, 2000, at the Collège de France. On page xv we reproduce the original statement of the Directors and the Scientific Advisory Board. John Tate and Michael Atiyah each spoke about the Millennium Prize Problems: Tate on the Riemann Hypothesis, the Birch and Swinnerton-Dyer Problem, and the **P** vs **NP** problem; Atiyah on the Existence and Uniqueness Problem for the Navier–Stokes Equations, the Poincaré Conjecture, and the Mass Gap problem for Quantum Yang–Mills Theory. In addition, Timothy Gowers gave a public lecture, "On the Importance of Mathematics". The lectures — audio, video, and slides — can be found on the CMI website: `www.claymath.org/millennium`.

The present volume sets forth the official description of each of the seven problems and the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

The editors gratefully acknowledge the work of Candace Bott (editorial and project management), Sharon Donahue (photo and photo credit research), and Alexander Retakh (TeX, technical, and photo editor) for their care and expert craftsmanship in the preparation of this manuscript.

James Carlson, Arthur Jaffe, and Andrew Wiles