CONTENTS

INTRODUCTION

§1 What is this book, and who is it for? 9
§2 Our math circles: Where is this book from? 10
§3 Some special features of this book 11
§4 Planning a session 12
§5 Running the session 13
§6 Problem-solving techniques 15
§7 Jump right in! 15

BULGARIAN SOLITAIRE

§1 Introduction 17
§2 Exploring the game 18
→ Jump right in!
 §2.1 The value of giving time to data collection 20
 §2.2 How to collect the data 21
 §2.3 A full data set leads to questions 22
 §2.4 Integer partitions 23
 §2.5 The fixed position case 24
 §2.6 The number of cycles 25
§3 The tilted model (optional) 26
 §3.1 The game settles down 27
 §3.2 The fixed positions appear 30
 §3.3 The cycles appear 30
§4 Further investigations 33

SONA

§1 Introduction 35
§2 Some simplifying assumptions about sona 37
→ Jump right in!
§3 Playing with graph paper 38
§4 Some counting questions 41
   §4.1 Number of turns 41
   §4.2 Number of intersections 42
   §4.3 How to approach counting questions 43
§5 The number of lines needed 43
§6 Exploring why the gcd shows up 44
   §6.1 Examine square arrays 45
   §6.2 Figure out what happens when we append or remove a square 46
   §6.3 Use the previous step to reduce the problem to a smaller one 48
   §6.4 Realize that we are describing the gcd 48
§7 Further investigations 51
Handout: All about Sona! 52

NIM & JIM 59

§1 Introduction 59
§2 Warm-up games: Don't Be Greedy and Two-Pile Nim 60
   §2.1 Don't Be Greedy 60
   → Jump right in! 64
   §2.2 Two-Pile Nim 64
   §2.3 Optional: One or One or One of Each 65
   §2.4 Optional: Describing a winning strategy 66
§3 The Game of Jim 67
   §3.1 Facilitator’s guide to the handout for this chapter 69
Handout: The Game of Jim 70
§1 Finishing up: Back to Nim 73
§2 Extensions/Challenges 74
§3 A methodical analysis and solutions to the misère versions 74
   §3.1 Don't Be Greedy, revisited 75
   §3.2 Two-Pile Nim, revisited 77
   §3.3 One or One or One of Each, revisited 79
   §3.4 Three-Pile Nim, revisited 81
§4 An alternate sequencing of these games 82
§5 Further investigations 83
SPLITTING TO MARS

§1 Introduction 85
§2 Get acquainted with the idea of splitting 86
→ Jump right in! 88
§3 Move to Venn diagrams 88
§4 Exploration and practice with Venn diagrams 90
§5 Some discussion questions about splitting 91
§6 Focus on scientists with just one specialty 93
§7 The case when no scientists have just one specialty 94
§8 A few more configurations and a conclusion 96
§9 Further investigations 97
Handout: Splitting to Mars 100

HYPERBINARY NUMBERS AND THE CALKIN–WILF TREE

§1 Introduction 107
→ Jump right in! 108
§2 Motivation: Making numbered lists 108
§3 Hyperbinary numbers 110
  §3.1 Binary representations 110
  §3.2 Hyperbinary representations 111
  §3.3 The pattern for odd numbers 113
  §3.4 The pattern for even numbers 115
  §3.5 The patterns determine the sequence 116
§4 The Calkin–Wilf tree 117
§5 Putting the two investigations together 119
§6 Further investigations 122
§7 Appendix: Proofs of the remaining claims (optional) 123
Handout: Hyperbinary numbers 125

GLOSSARY 129

§1 Binary numbers 129
§2 Greatest common divisor 130