Preface

> He who despises Euclidean Geometry is like a man who, returning from foreign parts, disparages his home.
>
> *H. G. Forder.*

The mathematics curriculum in the secondary school normally includes a single one-year course in plane geometry or, perhaps, a course in geometry and elementary analytic geometry called tenth-year mathematics. This course, presented early in the student's secondary school career, is usually his sole exposure to the subject. In contrast, the mathematically minded student has the opportunity of studying elementary algebra, intermediate algebra, and even advanced algebra. It is natural, therefore, to expect a bias in favor of algebra and against geometry. Moreover, misgüided enthusiasts lead the student to believe that geometry is "outside the main stream of mathematics" and that analysis or set theory should supersede it.

Perhaps the inferior status of geometry in the school curriculum stems from a lack of familiarity on the part of educators with the nature of geometry and with advances that have taken place in its development. These advances include many beautiful results such as Brianchon's Theorem (Section 3.9), Feuerbach's Theorem (Section 5.6), the Petersen–Schoute Theorem (Section 4.8) and Morley's Theorem (Section 2.9). Historically, it must be remembered that Euclid wrote for mature persons preparing for the study of philosophy. Until our own century, one of the chief reasons for teaching geometry was that its axiomatic method was considered the best introduction to deductive reasoning. Naturally, the formal method was stressed for effective educational purposes. However, neither ancient nor modern geometers have hesitated to adopt less orthodox methods when it suited them. If trigonometry, analytic geometry, or vector methods will help, the geometer will use them. Moreover, he has invented modern techniques of his own that are elegant and powerful. One such technique is the use of *transformations* such as rotations, reflections, and dilatations, which provide shortcuts in proving certain theorems and also relate geometry to crystallography and art. This "dynamic" aspect of geometry is the subject of Chapter 4. Another "modern" technique is the method of *inversive* geometry, which deals with points and circles, treating a straight line as a circle that happens to pass through "the point at infinity". Some flavor of this will be found in Chapter 5. A third technique is the method of *projective* geometry, which disregards all considerations of distance and angle but stresses the analogy between points and lines (whole infinite lines, not mere segments). Here not only are any two points joined by a line, but any two lines meet at a point; parallel lines are treated as lines whose common point happens to lie on "the line at infinity". There will be some hint of the content of this subject in Chapter 6.

Geometry still possesses all those virtues that the educators ascribed to it a generation ago. There is still geometry in nature, waiting to be recognized and appreciated. Geometry (especially projective geometry) is still an excellent means of introducing the student to axiomatics. It still possesses the esthetic appeal it always had, and the beauty of its results has not diminished. Moreover, it is even more useful and necessary to the scientist and practical mathematician than it has ever been. Consider, for instance, the shapes of the orbits of artificial satellites, and the four-dimensional geometry of the space-time continuum.

Through the centuries, geometry has been growing. New concepts and new methods of procedure have been developed: concepts that the student will find challenging and surprising. Using whatever means will best suit our purposes, let us revisit Euclid. Let us discover for ourselves a few of the newer results. Perhaps we may be able to recapture some of the wonder and awe that our first contact with geometry aroused.

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