

Preface to the First Edition

The purpose of this book is to introduce and develop some of the important and beautiful elementary mathematics needed for rational analysis of various gambling and game activities. While the only formal mathematics background assumed is high school algebra, some enthusiasm for and facility with quantitative reasoning will also serve the reader well. The book will, I hope, be of interest to:

1. Bright high school students with a good mathematics background and an (often related) interest in games of chance.
2. Students in elementary probability theory courses who might appreciate an informal supplementary text focusing on applications to gambling and games.
3. Individuals with some background in mathematics who are interested in some common and uncommon elementary game-oriented applications and their analysis.
4. That subset of the numerate gambling and game-playing public who would like to examine the mathematics behind games they might enjoy and who would like to see mathematical justification for what constitutes “good” (rational) play in such games.

One guiding principle of the book is that no mathematics is introduced without specific examples and applications to motivate the theory. The mathematics developed ranges from the predictable concepts of probability, expectation, and binomial coefficients to some less well-known ideas of elementary game theory. A wide variety of standard games are considered

along with a few more recently popular and unusual activities such as backgammon and state lotteries. Generally it is not the play of the game, but the reasoning behind the play that provides the emphasis.

Some readers may be temporarily slowed down or frustrated by the omission of detailed rules for a few of the games analyzed. There are two important reasons for such omissions. First, I believe that initial exposure to a new game should be acquired through experience rather than a formal study of rules. Thus, I would encourage readers unfamiliar with backgammon to find knowledgeable partners and to learn by playing. In a classroom setting, the promise of a class tournament is an excellent way to encourage students to master the rules before any mathematical analysis is engaged in. Secondly, the book is not intended to be a “how-to” document and I would urge readers interested in detailed rules and neatly packaged advice for backgammon, bridge, poker, or horse racing to consult one of the many specialized books on the particular topic (see the Bibliography). I give little “advice” that is not justified by prior mathematical analysis. In most cases it would be unreasonable to claim that this book will directly improve one’s gaming ability, though I would hope that the insights provided into rational analysis and play will have some spin-off in that direction. Exceptions to this disclaimer may be backgammon, which is treated in some detail, and the various gambling house games, for which it is made quite clear (blackjack being a possible exception) that the optimal strategy is abstinence.

After an historical and literary initial chapter, subsequent chapters (except for the last) each include 12 carefully selected exercises. These game-related questions are not primarily intended as drill problems, but rather for readers genuinely interested in expansion of ideas treated in the text. Some of the later exercises for each chapter may be quite challenging and some questions are open-ended, with emphasis on analysis and explanation rather than numerical answers. Chapter 2 provides the foundation for most of what follows, and should be mastered. The first two sections of Chapter 4 are needed also for Chapter 5. Otherwise chapters are, with minor exceptions, independent of one another. Sections referring to particular games can be included or omitted according to taste. The Bibliography presents, among other things, full reference information for books and articles referred to in abbreviated form in the text.

No serious judgments on the morality of gambling are intended in the book, though readers will no doubt draw inferences from the tone and tenor of my remarks at various points. While it should be clear that I am enthusiastic about games of chance and skill, I hope it is equally clear that I

am even more enthusiastic about the mathematics behind such games. It is this latter enthusiasm that I hope will be transferred to the reader.

Numerous institutions and individuals have helped me in shaping this book. Lake Forest College and three separate groups of students took their chances with me in the book's initial 3 years in an experimental freshman level course from which the final text has evolved. The Division of Social Sciences and Humanities at California Institute of Technology provided office space and technical assistance. Members of the New Mathematical Library subcommittee of the Mathematical Association of America have provided exceedingly thorough and helpful comments on the manuscript. I would also like to thank Bruce Cain, Charles Maland, and Kathryn Dohrmann for their valuable suggestions at various stages in the writing process. Finally, I am indebted to Gertrude Lewin for her typing of the first draft and Barbara Calli for her lovely artwork and typing of the final draft.

Preface to the Second Edition

The *mathematics* of games and gambling, for our pedagogical purposes, has not changed dramatically in the twenty-five years since the first edition of this book appeared. Probability theory continues to play a major role, game analyses still depend on expected value calculations together with computer simulations and game theory models for human behavior still strive mightily to capture an elusive reality.

Dramatic changes have occurred, however, in the games themselves, the way they are played and the public awareness of both game theory and gambling in its many forms. With the remarkable development of the Internet, we now have online gaming and wagering on virtually every reasonably well-known game and sports event. Casinos aggressively and creatively develop new games and increasingly sophisticated slot machines. A surprising collaboration between cable television and casino poker rooms has led to a popular explosion in a version of poker known as Texas Holdem. Even game theory itself has had its media moments with the awarding of several Nobel Prizes to game theorists and the making of a celebrated motion picture (“A Beautiful Mind”) about one of them.

This new edition responds to these changes in several ways. Sections have been added on the mathematical underpinnings of video poker, on aspects of bluffing in poker and on sports betting. The exercises have been augmented and slightly expanded to support some of the new material. I have also added web links to information on a number of game topics as well as to sites and applets for playing certain games against the computer. While some of these links may be ephemeral and also subject to invasion by commercial interests, I have tried to be selective in my choices and I believe that the value of such links for instructors and readers far outweighs these

risks. A listing of these web links is included at the end of the bibliography section.

Changes in content and in how the world plays its games notwithstanding, the goals of the book remain the same. The power and the beauty of both the mathematics and its application have provided the prime motivation for my efforts. I hope that readers will come to share my view that mathematics is the most fascinating game of them all.

Ed Packel
Lake Forest, Illinois
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Acknowledgements and Dedication

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Two mathematicians deserve special thanks. Harold Boas employed eagle-eyed editing and mathematical meticulousity in offering many valuable and insightful comments. And Richard Guy made many worthwhile suggestions, mathematical and otherwise, while performing powerful pyro \TeX nic in formatting many of the book's tables and formulas.

Finally, I dedicate this book to my wife, Kathryn Dohrmann—with gratitude for her editing, encouragement, enthusiasm and everything else.