

Contents

Preface	xi
Scott Sheffield and Thomas Spencer	
Introduction	1
David C. Brydges	
Lectures on the Renormalisation Group	7
Acknowledgment	9
Lecture 1. Scaling limits and Gaussian measures	11
1.1. Introduction	11
1.2. Theoretical physics	13
1.3. Some results	15
1.4. Gaussian measures on \mathbb{R}^Λ	15
1.5. Example: One dimension	18
1.6. Local and global functions	19
1.7. Example: Particles on a lattice	21
1.8. The importance of the partition function	22
1.9. Appendix. Green's functions	24
Lecture 2. Renormalisation group in hierarchical models	27
2.1. Massless Gaussian measure	27
2.2. Finite range decompositions	27
2.3. Motivation	29
2.4. The renormalisation group and hierarchical models	30
2.5. Hierarchical models	31
2.6. The formal infinite volume limit and trivial fixed point	34
2.7. Analysis	35
2.8. Expanding directions, relevant operators	35
2.9. Trivial fixed point	36
2.10. Appendix. Stable manifold theorem	37
Lecture 3. Example: the hierarchical Coulomb gas	41
3.1. Example: Hierarchical Coulomb gas	41
3.2. Finite volume	46
3.3. Fractional charge observable and confinement	47
3.4. Appendix. Notes on the rigorous renormalisation group	49
Lecture 4. Renormalisation group for Euclidean models	53
4.1. Euclidean lattice and the dipole model	53
4.2. The initial I_0, K_0	54
4.3. The basic scaling mechanism	55
4.4. Coordinates (I_j, K_j)	55
4.5. Euclidean replacement for Lemma 2.14	57

Lecture 5. Coordinates and action of renormalisation group	61
5.1. Euclidean replacement for Lemma 2.14 continued	61
5.2. Formulas for \tilde{I}, J .	64
Lecture 6. Smoothness of (RG)	67
6.1. Choice of spaces and smoothness of (RG)	67
6.2. Norms	71
6.3. Open problems	78
6.4. Appendix. Geometry and counting lemmas	78
6.5. Appendix. Proof of Theorem 6.14	82
Bibliography	91
Alice Guionnet	
Statistical Mechanics and Random Matrices	95
Introduction	97
1. Motivations	98
2. The different scales; typical results	104
Lecture 1. Wigner matrices and moments estimates	109
1. Wigner's theorem	109
2. Words in several independent Wigner matrices	118
3. Estimates on the largest eigenvalue of Wigner matrices	120
Lecture 2. Gaussian Wigner matrices and Fredholm determinants	123
1. Joint law of the eigenvalues	123
2. Joint law of the eigenvalues and determinantal law	124
3. Determinantal structure and Fredholm determinants	126
4. Fredholm determinant and asymptotics	127
Lecture 3. Wigner matrices and concentration inequalities	131
1. Concentration inequalities and logarithmic Sobolev inequalities	131
2. Smoothness and convexity of the eigenvalues of a matrix and of traces of matrices	135
3. Concentration inequalities for random matrices	139
4. Brascamp-Lieb inequalities; applications to random matrices	141
Lecture 4. Matrix models	147
1. Combinatorics of maps and non-commutative polynomials	149
2. Formal expansion of matrix integrals	153
3. First order expansion for the free energy	160
4. Discussion	167
Lecture 5. Random matrices and dynamics	171
1. Free Brownian motions and related stochastic differential calculus	172
2. Consequences	179
3. Discussion	182
Bibliography	185

Richard Kenyon	
Lectures on Dimers	191
1. Overview	193
1.1. Dimer definitions	193
1.2. Uniform random tilings	194
1.3. Limit shapes	196
1.4. Facets	198
1.5. Measures	199
1.6. Other random surface models	200
2. The height function	200
2.1. Graph homology	200
2.2. Heights	201
3. Kasteleyn theory	202
3.1. The Boltzmann measure	203
3.2. Gauge equivalence	203
3.3. Kasteleyn weighting	203
3.4. Kasteleyn matrix	204
3.5. Local statistics	205
4. Partition function	206
4.1. Rectangle	206
4.2. Torus	207
4.3. Partition function	209
4.4. Height change distribution	209
5. Gibbs measures	209
5.1. Definition	209
5.2. Periodic graphs	210
5.3. Ergodic Gibbs measures	210
5.4. Constructing EGMs	211
5.5. Magnetic field	211
6. Uniform honeycomb dimers	212
6.1. Inverse Kasteleyn matrix	213
6.2. Decay of correlations	213
6.3. Height fluctuations	214
7. Legendre duality	215
8. Boundary conditions	217
9. Burgers equation	218
9.1. Volume constraint	220
9.2. Frozen boundary	220
9.3. General solution	220
10. Amoebas and Harnack curves	222
10.1. The amoeba of P	222
10.2. Phases of EGMs	223

10.3. Harnack curves	224
10.4. Example	224
11. Fluctuations	226
11.1. The Gaussian free field	226
11.2. On the plane	227
11.3. Gaussians and moments	227
11.4. Height fluctuations on the plane	227
12. Open problems	229
Bibliography	229
G. Lawler	
Schramm-Loewner Evolution (SLE)	231
Introduction	233
Lecture 1. Scaling limits of lattice models	235
1. Self-avoiding walk (SAW)	235
2. Loop-erased random walk	239
3. Percolation	241
4. Ising model	242
5. Assumptions on limits	243
6. Exercises for Lecture 1	243
Lecture 2. Conformal mapping and Loewner equation	245
1. Important results about conformal maps	245
2. Half-plane capacity	247
3. Loewner equation	249
4. Maps generated by a curve	251
5. A flow on conformal maps	252
6. Doubly infinite time	253
7. Distance to boundary	254
8. Exercises for Lecture 2	255
Lecture 3. Schramm-Loewner evolution (SLE)	257
1. Definition	257
2. Phases	259
3. Dimension of the path	260
4. Cardy's formula	261
5. Conformal images of SLE	262
6. Exercises for Lecture 3	264
Lecture 4. SLE_κ in a simply connected domain D	265
1. Drift and locality	265
2. Girsanov	266
3. The restriction martingale	267
4. (Brownian) boundary bubbles	268
5. Brownian loop measure	270
6. The measure $\mu_D(z, w)$ for $\kappa \leq 4$	271
7. Exercises for Lecture 4	272

Lecture 5. Radial and two-sided radial SLE_κ	275
1. Example: SAW II	275
2. Radial SLE_κ	277
3. Another definition	279
4. Radial SLE_κ in a smaller domain	280
5. Two-sided radial	281
6. Exercises for Lecture 5	282
Lecture 6. Intersection exponents	283
1. One-sided	283
2. Two-sided	286
3. Nonintersecting SLE_κ	287
4. Radial exponent and SAW III	288
5. Exercises for Lecture 6	290
Tables for reference	291
Bibliography	293
Wendelin Werner	
Lectures on Two-dimensional Critical Percolation	297
Overview	299
Lecture 1. Introduction and tightness	301
1. 2D percolation	301
2. Notations and prerequisites	302
3. Russo-Seymour-Welsh	304
4. First consequences	306
First exercise sheet	309
Lecture 2. The Cardy-Smirnov formula	313
1. Preliminaries	313
2. Smirnov's theorem	315
Lecture 3. Convergence to SLE(6)	321
1. Our goal	321
2. Hand-waving argument	322
3. Tightness	323
4. Loewner chains	323
5. Capacity increases	324
6. Side-remark concerning the proof of Cardy-Smirnov formula	326
7. Identifying continuous martingales	326
8. Recognizing SLE(6)	327
9. Take-home message	328
Second exercise sheet	331
Lecture 4. SLE(6) computations	333
1. Radial SLE	333
2. Relation between radial and chordal SLE(6)	334

3. Relation to discrete radial exploration	335
4. Exponent computations	336
Lecture 5. Arm exponents	341
1. Some notations	341
2. One-arm exponent	341
3. Four-arms exponent	343
4. Other exponents and bibliographical remarks	349
Lecture 6. Near-critical percolation	351
1. Correlation length	351
2. Outline of the proof	352
3. A priori estimates	353
4. Arm separation	353
5. Using differential inequalities	354
6. Concluding remarks	358
Bibliography	359