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Introduction

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1. Introduction

This volume contains lecture notes from the nine lecture series on mapping class groups and moduli spaces of Riemann surfaces that were given at the Park City Mathematics Institute in July, 2011. These lecture series were directed at graduate students and researchers interested in these topics.

Let us recall how these two topics that were so central to this PCMI workshop are connected. The *mapping class group* of a closed, connected, oriented surface S is the group of isotopy classes of its orientation preserving diffeomorphisms. The *Teichmüller space* of that surface is the set of all isotopy classes of its conformal structures. If we assume the genus g of S to be positive, then this is a contractible complex manifold on which the mapping class group of S acts properly discontinuously, with a subgroup of finite index acting freely. The fact that the orbit space of this action may be understood as the moduli space $\mathcal{M}_g$ of Riemann surfaces of genus g accounts for the close relationship between the mapping class group and the topology of $\mathcal{M}_g$.

We can take this further: a compact Riemann surface is in a unique way a smooth complex-projective curve and so $\mathcal{M}_g$ is also the moduli space of smooth projective curves of genus g. With that interpretation it acquires a much finer structure, namely that of an algebraic variety (or more precisely, of a Deligne-Mumford stack, at least when $g \geq 2$) defined over the rational number field. As a result, moduli spaces of Riemann surfaces are fundamental objects of study in topology, complex analysis, and algebraic geometry; they can be studied from each of these points of view.

Each of the communities studying moduli spaces of curves — geometric topologists, algebraic topologists, complex analysts, algebraic and arithmetic geometers — generates its own set of problems and a set of techniques for resolving them. These problems and techniques are quite divergent. The goal of the PCMI summer school was cross-pollination: to educate students and researchers from these sometimes disparate communities in the basic problems and techniques of the others. Our premise was that interactions between these groups are fruitful in both the short and long term. This volume has the same goals; we believe that the authors have achieved this goal.

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The lectures in this volume cover a broad range of topics, from introductory to advanced material. Several of the articles cross disciplinary boundaries. They are generally arranged from the more introductory to the more advanced. That said, such an ordering of the articles will depend upon one's background and taste. We encourage students to explore all of the lectures to develop a broad view of the subject. We encourage established researchers to get out of their comfort zone and explore the lectures in this volume that are not in their immediate areas of expertise. There they may find new ideas, problems and techniques.