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Introduction

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The 2015 Park City Mathematics Institute program on “Geometry of moduli spaces and representation theory” was devoted to a combination of interrelated topics in algebraic geometry, topology of algebraic varieties and representation theory.

Geometric representation theory is a young but fast developing research area at the intersection of the those subjects. An early profound achievement was the formulation, in the late 70’s, of Kazhdan and Lusztig’s famous conjecture on characters of highest weight modules over a complex semi-simple Lie algebra, and its subsequent proof by Beilinson–Bernstein and Brylinski–Kashiwara. Two remarkable features of this proof have inspired much of subsequent development: intricate algebraic data turned out to be encoded in topological invariants of singular geometric spaces, while proving this fact required deep general theorems from algebraic geometry. The topological invariants in question have to do with *intersection cohomology* of Schubert varieties, while the key algebro-geometric result used in the proof is a generalization of Weil’s conjecture by Beilinson, Bernstein and Deligne involving perverse sheaves.

The geometric spaces appearing in the Kazhdan–Lusztig conjectures are closed subvarieties in the flag variety, a homogeneous space which is a basic ingredient in the theory of algebraic groups. A later major direction in geometric representation theory, shaped by contributions of Lusztig, Nakajima and others, develops a similar relation between representation theory and moduli spaces of linear algebra data (quiver varieties).

More intricate geometric objects have entered the subject with the emergence of the geometric Langlands program. This direction, pioneered by Beilinson and Drinfeld in the 90’s, is partly inspired by Langlands’ conjectural nonabelian reciprocity laws from number theory. In the last decade, Kapustin and Witten have discovered its close connection to S-duality in quantum field theory. While employing some of the techniques of Kazhdan-Lusztig theory, geometric Langlands duality deals with more sophisticated geometric spaces, such as the moduli space (or stack) of principal bundles on a complete algebraic curve and its local counterpart, the affine Grassmannian, also known as the loop Grassmannian. A large part of the PCMI program was devoted to introducing this circle of ideas.

Another focus of the program was on some aspects of enumerative algebraic geometry. Recent progress in that area has been increasingly bringing to light the role of Lie theoretic structures in problems such as calculation of (equivariant) quantum cohomology, *K*-theory etc. Although the motivation and technical background of these constructions is quite different from that of geometric Langlands duality, both theories deal with topological invariants of moduli spaces of maps from a target of complex dimension one. Thus they are at least heuristically related, while several recent works indicate possible strong technical connections.

The goal of the program was to provide an introduction to these areas of active research and promote interaction between the two related directions. Our hope is that this will help to write a new chapter in the glorious history of the interaction between representation theory and algebraic geometry. Just as *D*-modules, perverse sheaves and the generalizations of Weil's conjecture have become standard tools in studying many algebraic questions in representation theory, we hope that keys to resolving other outstanding questions may lie in the recent techniques of enumerative algebraic geometry

The program included minicourses by Alexander Braverman, Mark de Cataldo, Victor Ginzburg, Davesh Maulik, Hiraku Nakajima, Xinwen Zhu, Zhiwei Yun, and Clay Scholars Ngô Bảo Châu and Andrei Okounkov. This volume contains contributions by Mark de Cataldo, Hiraku Nakajima, Ngô Bảo Châu, Andrei Okounkov, Xinwen Zhu and Zhiwei Yun.