Preface

Mathematical problems can be appealing for a variety of reasons, but we are especially fond of problems that have a big surprise factor. The surprise can take different forms. Our previous problem book, *Which Way Did the Bicycle Go?* [**82**], highlighted a problem in bicycle geometry where the surprise was a bit unusual: the fact that Sherlock Holmes made a serious error in logical reasoning. The present book starts with another bicycle problem, one for which the surprise is that a bicycle can pretend to be a unicycle.

We have selected the 105 problems in this collection with an eye to the surprise factor. Almost all of these problems appeared in the Problem of the Week program at Macalester College. After his retirement from Macalester, Stan Wagon kept the program going via a mailing list.

The surprises come in a variety of forms. For some problems the goal is to determine an integer, but at the start one would have no idea if the size of the answer is near 10, 100, or $10^{10^{100}}$. A remarkable one of this type is a cake-slicing problem (Problem 21) that everyone who guesses at the answer gets wrong. In this case, the answer is quite different from what one would expect. Three other examples are Problems 37, 38, and 91.

An important aspect of problem posing is getting the statement exactly right. A notorious probability problem (Problem 49) concerns a parent who has two children, one of whom is a son born on a Tuesday. This particular problem has generated an enormous amount of discussion, as the result appears to be paradoxical. But the key is how the problem is posed, and we have done so in a way that makes it possible to clear up the mystery.

A different type of paradox, one that highlights the difference between large finite sets and infinite sets, is discussed in Problem 79. That problem shows the importance of the choice of fundamental axioms for mathematics.

Two problems (Problems 90 and 96) show that contest problems can provide a big surprise to the writers of the contest, as the problems turned out to be much more delicate than the designers intended.

Situations where π appears unexpectedly have always fascinated mathematicians and the general public. There are the classics such as the Buffon needle problem with its probability of $2/\pi$ and the sum of the reciprocals of the square integers, which Euler proved to be $\pi^2/6$. For one problem in this collection π makes an appearance that is just as, or more, surprising than the two classical examples.

Problem 95, about the ways in which polynomials change as they move from left to right, shows that new discoveries can be made involving very elementary notions.

As is traditional in mathematics, some problems are based on ideas from physics. Trajectories of projectiles are considered in Problems 61 and 62. Problem 66 concerns the center of mass of a partially filled glass of water, and Problem 105 deals with collisions between sliding blocks. Also, again as is traditional, problems lead to further problems. Problem 81 is a not-too-difficult problem about integers. But it leads to a much more surprising real-number version, Problem 82, whose solution uses entirely different methods. And many problems here lead naturally to open questions.

Sometimes popular culture or games can lead to interesting problems. Five of that sort are related to the games of poker (Problem 54) and Bingo (Problem 56), the culture of crossword puzzles (Problems 92 and 93), and a puzzle feature of the first Harry Potter book (Problem 94).

Some of our problems have short and concise solutions, while others are quite complicated. Almost all can be done by hand, but occasionally help from computers is needed, and we view such a tool as indispensable for investigations into many problems. While almost all of our problems are typical of the genre, in that the solutions use standard mathematical tools and are not overly complicated, we have included a few that really are research projects (e.g., Problem 1). In a couple of cases, a rigorous proof requires a result that might not be well known and we have included the full details.

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Notation. We use the following notation. $\mathbb{N} = \{0, 1, 2, \ldots\}$, the natural numbers; $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the integers. $[n]$ denotes $\{1, 2, \ldots, n\}$. A permutation is denoted as a row of numbers, such as $3\,1\,4\,2$ for $\binom{1\,2\,3\,4}{3\,1\,4\,2}$, which is the cycle $1 \to 3 \to 4 \to 2 \to 1$. This cycle is also denoted $(1\,3\,4\,2)$. An approximation to a real number is denoted by $\approx$, as in $\pi \approx 3.142$.