Preface

The book that the reader is about to open is a collection of articles written in memory of Boris Dubrovin. By it, the authors express their admiration for his remarkable personality and for the contribution he made to mathematical physics. For many, he was a friend, a colleague, for some an inspiring mentor and teacher. For all who knew him, he was a man in joyful love for life, in all its manifestations.

Boris passed away on March 19, 2019, at the age of 68 (he was born on April 6, 1950), after a long and courageous battle against a rare and terrible disease (ALS).

Boris graduated from Moscow State University. He was a student of Sergey P. Novikov. In 1974 they created the “Periodic Inverse Scattering transform” for the Korteweg-de Vries (KdV) equation.

Boris’s first steps in mathematics came at a remarkable time of the birth of a new field of mathematical physics. After the discovery by Kruskal and Zabuski of the infinite number of integrals of the KdV equation the “Inverse Scattering Transform” was created in the famous work (1967) by Gardner, Green, Kruskal, Miura, clarified later by Lax. After works by Zakharov-Shabat and Ablowitz-Kaup-Newal-Segur it became clear that a wide class of fundamental nonlinear equations of mathematical physics are within the reach of analytic methods – and not only the computer simulations.

The mathematics needed was already in the ready-to-use state in the case of rapidly decreasing initial data, but far from being effective enough in the case of periodic problems. A breakthrough in periodic problems was done by Novikov (1974) who found the “finite-gap” solutions of the KdV equation and proved that stationary solutions to higher KdV equations are finite-gap Schrodinger potentials (whose degenerate limit are rapidly decreasing reflectionless potentials). Dubrovin’s first result was an “inverse theorem”: finite-gap potentials are stationary solutions of higher KdV equations. The proof was based on ideas and methods of classical algebraic geometry. Establishing new unexpected connections between seemingly unrelated areas of mathematics was a characteristic feature of Boris’s scientific style.

Periodic (2+1)-dimensional problems were initiated by Krichever (1976) and developed by Dubrovin, Krichever, Novikov for 2D Schrodinger operators (1976). A lot of famous mathematicians developed periodic soliton theory since 1975 – Lax, McKean, Marchenko, and their schools. Its and Matveev interacted with Novikov and Dubrovin since 1974 and made significant contributions.

Among Boris’s numerous works on the finite gap theory are the spectral theory of finite-gap operators with matrix coefficients and his work with Natanzon on the periodic solutions to the famous sine-Gordon equation. The latter problem turned out to be very difficult and its solution was completed by Novikov and Grinevich in 2003.

It is hard to overestimate the influence of his paper "Hydrodynamics of weakly deformed soliton lattices. Differential geometry and Hamiltonian theory", written together with S. Novikov, where the foundations of the Hamiltonian theory of the Whitham perturbation theory of finite gap solutions were laid and connections with the theory of n-orthogonal curvilinear coordinate systems, or flat diagonal metrics were established. For more than a century since the famous work of Dupin and Binet, the problem of the construction of such systems was one of the most important problems of differential geometry. Treated as a classification problem, it was mainly solved by G. Darboux at the beginning of the 20th century.

These connections were central to the most famous concept introduced by Boris at the beginning of the 90s: the notion of the Frobenius manifolds which turned out to be a proper geometric language for the celebrated Witten-Dijkgraaf-Verlinde-Verlinde equations introduced in an attempt to classify topological quantum field models. From the geometrical point of view, an identification of a space of the deformation parameters of such theory with the ring of primary fields can be seen as a compatible multiplication on the tangent space that has the structure of a Frobenius algebra. Boris observed that the WDVV equations are equivalent to the flatness condition of a certain connection with a "spectral" parameter on the tangent space. The most important Boris's addition to the WDVV equations was the requirement of quasi-homogeneity of their solutions, which became a part of the axiomatics of Frobenius manifolds. Under the assumption of quasi-homogeneity, the WDVV equations can be identified with the equations of isomonodromic deformations of the ordinary equation with rational coefficients. Thus, quasi-homogeneous solutions of the WDVV equations are uniquely determined by the monodromy data of the corresponding equation. In essence, this result is the answer to the problem of classifying topological quantum field theories.

Connections of the theory of Frobenius manifolds with the classical theory of isomonodromy deformations have proven to be beneficial for both sides. Motivated by the study of algebraic Frobenius manifolds, Boris Dubrovin and M. Mazzocco proposed a new approach to the problem of classification of algebraic solutions of the Schlesinger equations. In particular, they classified the algebraic solutions of a certain case of the Painleve-VI equation. Later, Dubrovin - Mazzocco's approach was extended by O. Lysovyj et al. to a complete classification of algebraic solutions of the general Painleve-VI equation.

Frobenius manifolds play a central role in the mathematical formulation of the mirror symmetry, which is one of the most influential ideas brought to mathematics by theoretical physics. At the end of 90s Boris proved that quantum cohomology of Fano varieties produce a Frobenius manifold and connect it with the Frobenius manifold produced by the Saito theory of hypersurface singularity.

The classification of semi-simple Frobenius manifolds which are in one-to-one correspondence dispersionless limits of integrable PDE was a starting point of Boris approach towards the classification problem of integrable evolution equations. In a series of works, Boris and Youjin Zhang were able to reconstruct, using Frobenius manifolds, the integrable dispersive PDEs from the integrable systems of hydrodynamic type. They have shown that a particular solution to those PDEs matches the generating function of the intersection numbers of given cohomological field theory (CohFTs). An important application of the Dubrovin-Zhang approach to CohFT is the completion of the Gromov–Witten invariants of the complex projective line in terms of a new integrable hierarchy, the so-called extended Toda lattice (jointly with G. Carlet and Y. Zhang). This work has completed the analysis initiated earlier by T.Eguchi et al and by A. Okounkov and R. Pandharipande. Later, B. Dubrovin, in collaboration with D. Yang and M. Bertola, developed the concept of topological ODEs, in order to compute in a simple and very efficient way the correlators of cohomological field theories and random matrices.

Another important achievement of Boris Dubrovin is related to the universality of the critical behavior of solutions of Hamiltonian PDEs, developed together with C. Klein, A. Moro, and T. Grava. The study of properties of solutions of systems of nonlinear Hamiltonian PDEs with slowly varying initial conditions gave rise to the remarkable discovery of the phenomenon of the universality of the behavior of a generic solution at the point of phase transition from regular to the oscillatory regime.

Nowadays, about 25 years after the introduction of Frobenius manifolds by Dubrovin, one finds in Google Scholar many thousands of citations on this subject.

Boris became a full Professor of mathematics at Moscow State University in 1988. He was elected as Distinguished Professor of Mathematical Physics at Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste, Italy, in 1993 and he served as director of the Mathematical Physics Group and the Mathematics area for several years. Since 2010 Boris has been the Director (and organizer) of the new Bogolyubov Laboratory Geometrical methods in Mathematical Physics of Moscow University. This Laboratory has become one of the most active mathematical centers in Moscow. He was a member of editorial boards of the Journal of High Energy Physics, Letters in Mathematical Physics, and Journal of Integrable Systems.

For his scientific achievements, Boris received a prize of Moscow Mathematical Society in 1976, together with A. Its and I. Krichever. He gave an invited talk at the International Congress of Mathematical Physicists at Swansea (1988), plenary talk at the European Congress of Mathematicians at Budapest (1996), invited talk at the International Congress of Mathematicians in Berlin (1998) and a plenary talk at the International Congress of Mathematical Physicists in Rio de Janeiro (2006).

The contributions to this collection of papers are split into two volumes. The volume titles – "Integrable Systems" and "Quantum Theories and Algebraic Geometry" – reflect the areas of main scientific interests of Boris. Chronologically, works of Boris may be divided into several parts: integrable systems, integrable systems of hydrodynamic type, WDVV equations (a.k.a Frobenius manifolds), isomonodromy equations (flat connections), quantum cohomology. The articles included in the first volume are those which are more or less directly devoted to these areas (primaryly with the first three listed above). The second volume contains articles devoted mostly to quantum theories, algebraic geometry and is less directly connected with the early interests of Boris. Of course, this splitting is by no means a clear-cut;

several papers are written by former coauthors of Boris and contain new results in related areas.

All of the authors of these volumes was influenced by Boris in some way during his prolific scientific life. This influence will grow with time.

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