Preface

Representation theory of Lie algebras, quantum groups, and related algebraic structures has become a central research area in mathematics, with numerous applications in mathematics and theoretical physics. Research topics include quantized enveloping algebras, quantum function algebras, Kac–Moody Lie algebras, Hecke algebras, canonical bases and crystal bases, vertex operator algebras, Hall algebras, A_∞-algebras, quivers, cluster algebras, Hopf algebras, and Khovanov–Lauda–Rouquier algebras. In particular, quantized Kac–Moody Lie algebras and related categorical and geometric constructions have taken on a leading role not only within Lie theory but also in other fields including combinatorics, group theory, number theory, integrable systems, low-dimensional topology and conformal field theory.

In 1985, the interaction of semisimple and Kac–Moody Lie algebras with integrable systems led Drinfeld and Jimbo to introduce a new class of algebraic objects known as quantized universal enveloping algebras (also known as quantum groups) associated with symmetrizable Kac–Moody Lie algebras. The abstract theory of integrable representations of quantum groups given by Lusztig illustrates the similarity between quantum groups and Kac–Moody Lie algebras.

Much of the theory of quantum groups, vertex algebras and Kac–Moody Lie algebras is based on results developed for algebraic groups and Lie groups. Much of the classical theory deals with algebraic groups over algebraically closed fields or local fields, but in recent decades, there has been an increasing focus on Kac–Moody groups. Recent developments include the introduction of a building-like structure for Kac–Moody groups, with applications to Kac–Moody symmetric varieties. The full impact of these results on algebraic groups, quantum groups and Kac–Moody Lie groups is yet to be realized.

The theory of canonical basis for quantum groups (developed independently by Lusztig and Kashiwara around 1990) has had a huge impact on the field and has provided important insights to representation theory at large. In particular, the canonical base at $q = 0$ (i.e., crystal base) provides a beautiful combinatorial tool to study the representations of quantum groups. Explicit realizations of crystal bases for finite-dimensional simple Lie algebras and affine Lie algebras led to interesting applications in combinatorics and mathematical physics. Furthermore, various geometric realizations of crystals have revealed natural connections with quiver varieties and affine Grassmannians. The theory of geometric crystals introduced by Berenstein and Kazhdan in 2000 has also brought new ideas to the area.

The canonical basis theory of Lusztig–Kashiwara has motivated the Khovanov–Lauda–Rouquier (KLR) categorification, whose striking applications include Broué's conjecture for symmetric groups, $\mathbb{Z}$-gradings on blocks of symmetric groups, quiver varieties, rational Cherednik algebras, and Reshetikhin-Turaev invariants.

The ıcanonical bases yielded geometric interpretations via flag varieties of classical type and they afforded the positivity property, as shown by Y. Li and collaborators. A general theory of quantum symmetric pairs has been developed by G. Letzter and others. The canonical bases can be generalized to more general quantum symmetric pairs. In the affine Kac–Moody case, the ıcanonical basis should be closely related to modular representation of classical type algebraic groups and quantum groups at roots of unity. The canonical basis of quantum symmetric pairs is also leading to a new KLR-type categorification and new diagrammatic algebras.

For quantum groups, Arkhipov, Bezrukavnikov, and Ginzburg (ABG) used the geometry of the nilpotent cone $\mathcal{N}$, whose coordinate algebra identifies with the cohomology ring, as a starting point for their seminal work. They established derived equivalences between the principal block for the quantum group, (equivariant) coherent sheaves on the Springer resolution of $\mathcal{N}$, and perverse sheaves on the loop Grassmannian. These equivalences of triangulated categories yielded a proof of Lusztig's character formula for quantum groups when $l > h$ (where l is the order of the root of unity). The ABG approach illustrated that to calculate characters of simple modules at the representation theoretic level one needs to examine the structure of the underlying tensor triangulated category at the derived level where the geometry becomes more transparent. An important piece of their work involved the lifting of the support variety theory for quantum groups to obtain the equivalence between the principal block and coherent sheaves on the Springer resolution.

In 2009, Kailash Misra, Daniel Nakano, and Brian Parshall established a network of Lie theorists in the southeastern region and proposed an annual regional workshop series of 3 to 4 days in Lie theory. The aim of these workshops was to bring together senior and junior researchers as well as graduate students to build and foster cohesive research groups in the region. Since that time, there have been thirteen successful workshops at North Carolina State University (2009, 2012, 2015 and 2023), the University of Georgia (2010, 2014 and 2018), the University of Virginia (2011 and 2016), the College of Charleston (2012 and 2021) and Louisiana State University (2013 and 2019). Each of these workshops included expository talks by senior researchers, invited talks on recent developments by mid-career and junior researchers, contributed talks by post-docs and senior graduate students along with afternoon discussion sessions, intended as an educational opportunity for graduate students and junior researchers interested in Lie theory. This book is a collection of papers contributed by some of the invited speakers at the workshops held during 2015–2021. The previous two volumes PSPUM 86 and PSPUM 92 in this series edited by Misra, Nakano, and Parshall covered the workshops during 2009–2011 and 2012–2014 respectively. We dearly miss Ben Cox, a member of the scientific committee and Brian Parshall, a founding member of the southeastern Lie theory network due to their sudden passing in September 2019 and January 2022 respectively and dedicate this volume to their memory.

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