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# Preface

Our lives and the universe barely work, but that's OK; it's amazing and great that they work at all. I think it has something to do with math, and especially real analysis, the theory behind calculus, which just barely works. Did you know that there are functions that are not the integral of their derivatives, and that a function with a positive derivative can decrease? But if you're a little careful you can get calculus to work. You'll see.

The theory is hard, subtle. After Newton and Leibniz invented the calculus in the late 1600s, it took puzzled mathematicians two hundred years, until the latter 1800s, to get the theory straight. The powerful modern approach using open and closed sets came only in the 1900s. Like many others, I found real analysis the hardest of the math major requirements; it took me half the semester to catch on. So don't worry: just keep at it, be patient, and have fun.

The applications in the calculus of variations are amazing, from computing optimal economic strategies to predicting the relativistic correction to Mercury's orbit.

This text is designed for students. It presents the theoretical intellectual breakthroughs which made calculus rigorous, but always with the student in mind. If a shortcut or some more advanced comments without proof provide better illumination, we take the shortcut and make the comments. The result is a complete course on real analysis that fits comfortably in one semester. Chapters 1–17 provide the theoretical core, which can for example be supplemented by Chapters 18–20 and 23–25 (Fourier Series) or by selections from Part V on the Calculus of Variations.

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