CONTENTS

1. Introduction 1
1.1. Measure on loops: history 1
1.2. Measure on loops: present work and recent developments 2
1.3. Results and layout 4
1.4. A list of commonly used notations 6
Chapter 2 6
Chapter 3 7
Chapter 4 8
Chapter 5 9
Chapter 6 9
Chapter 7 10

2. Preliminaries on generators and semi-groups 11
2.1. A second order ODE 11
2.2. One-dimensional diffusions 17
2.3. "Generators" with creation of mass 21

3. Measure on loops and its basic properties 29
3.1. Spaces of loops 29
3.2. Measures $\mu^{x,y}$ on finite life-time paths 30
3.3. The measure μ^* on unrooted loops 37
3.4. Multiple local times 41
3.5. Measure on loops rooted at the minimum 44
3.6. A generalization of the Vervaat's transformation 48
3.7. Restricting loops to a discrete subset 53
3.8. Measure on loops in case of creation of mass 55

4. Occupation fields of the Poisson ensembles of Markov loops 59
4.1. Inhomogeneous continuous state branching processes with immigration . 59
4.2. Occupation field 62
4.3. Isomorphism with the Gaussian free field 72

5. Decomposing paths into Poisson ensembles of loops 77
5.1. Gluing together excursions ordered by their minima 77
5.2. Loops represented as excursions and glued together 80
5.3. The case $\alpha = 1$ 87

6. Wilson's algorithm in dimension one 93
6.1. Description of the algorithm 93
6.2. The erased paths 95
6.3. Determinantal point processes $(\mathcal{Y}_\infty, \mathcal{Z}_\infty)$: Brownian case 97
6.4. Determinantal point processes $(\mathcal{Y}_\infty, \mathcal{Z}_\infty)$: general case 118

7. Monotone couplings for the point processes $(\mathcal{Y}_\infty, \mathcal{Z}_\infty)$ 119
7.1. Conditioning 119
7.2. Couplings 134

Bibliography 153