Abstract **(Generalized Configuration Spaces, i-Acyclic Spaces, Basic Spectral Sequences)**

This memoir presents a new approach to *generalized configuration spaces*

$$\Delta_{\leqslant \ell} M^m := \big\{(x_1, \ldots, x_m) \in M^m \;\big|\; \#\{x_1, \ldots, x_m \quad \leqslant \ell \;\; ,$$
$$\Delta_\ell M^m := \Delta_{\leqslant \ell} M^m \setminus \Delta_{<\ell} M^m \text{ and } F_m(M) := \Delta_m M^m$$

of a locally compact space M. The approach is two-fold.

The first part applies only to *i-acyclic* spaces, which class contains noncompact contractible spaces, and, if X is i-acyclic, contains also the open subspaces of X and the products $X \times M$ by any space M. For an i-acyclic space X, given $i, m \in \mathbb{N}$, the families of representations of symmetric groups $\big\{\mathcal{S}_{m-a} : H_{\mathrm{c}}^{i-a}(F_{m-a}(X)) \;\big|\; a \leqslant m\big\}$ and $\big\{\mathcal{S}_{m-a} : H_{\mathrm{c}}^{i-a}(X^{m-a}) \;\big|\; a \leqslant m\big\}$ are intertwined by a universal invertible matrix of induction functors in the category of FI-modules. This notable feature allows questions about the first family to be swapped to the second, where they are, a priori, more tractable. In this way we were able to express the character of the $\mathcal{S}_m$-module by a universal formula depending only on the quadruplet $(?, i, \ell, m)$ and the compact Betti numbers of X. The method also allowed us to extend Church's stability theorems to the families $\mathcal{D}_?(a) := \big\{\Delta_{?m-a} X^m\big\}_{m \geqslant a}$.

The second part describes a procedure which extrapolates cohomological properties of configuration spaces of i-acyclic spaces X to general topological spaces M. The main tool is the *basic spectral sequence* which converges to $H_{\mathrm{c}}(F_m(M))$ and whose first page terms are induced representations of the various $H_{\mathrm{c}}(F_{m-a}(M \times \mathbb{R}))$, for $0 \leqslant a \leqslant m$. The spectral sequence keeps track, page after page, of stability ranks, thereby giving estimates of them at the abutment. As an application, Church's representation stability theorems for the family $\big\{F_m(M)\big\}_m$ where M is an oriented manifold, are generalized to the families $\mathcal{D}_?(a) := \big\{\Delta_{?m-a} M^m\big\}_{m \geqslant a}$, where M is no more smooth. In particular, irreducible complex algebraic varieties, whether they are smooth or not, verify these generalizations.