

Chapter Five: Tower Squares

A Tower Square of order n is an $\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}$ table in which each row and each column consists of one copy of 1, two copies of 2, and so on, up to n copies of n . There is a trivial Tower Square of order 1 and six Tower Squares of order 2, as shown in the diagram below. For recreational purposes, we are primarily interested in Tower Squares of orders 3 and 4.

| | | |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 2 | 2 | 1 |

| | | |
|---|---|---|
| 2 | 1 | 2 |
| 2 | 2 | 1 |
| 1 | 2 | 2 |

| | | |
|---|---|---|
| 2 | 2 | 1 |
| 1 | 2 | 2 |
| 2 | 1 | 2 |

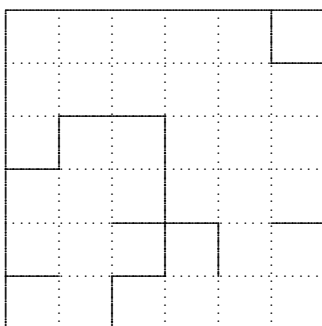
| | | |
|---|---|---|
| 2 | 2 | 1 |
| 2 | 1 | 2 |
| 1 | 2 | 2 |

| | | |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 1 |
| 2 | 1 | 2 |

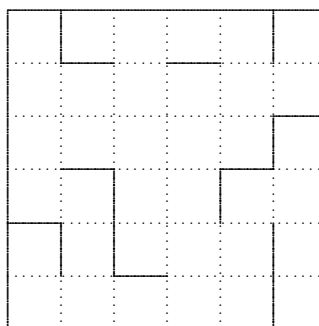
| | | |
|---|---|---|
| 2 | 1 | 2 |
| 1 | 2 | 2 |
| 2 | 2 | 1 |

In Problems 1 to 4, two squares separated by a segment must contain identical numbers, and two identical numbers in adjacent squares must be separated by a segment. Reconstruct the Tower Squares.

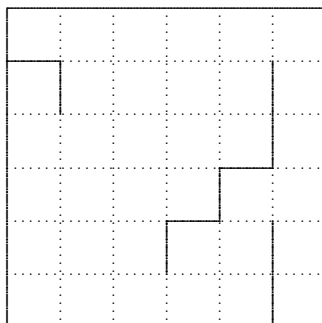
Problem 1



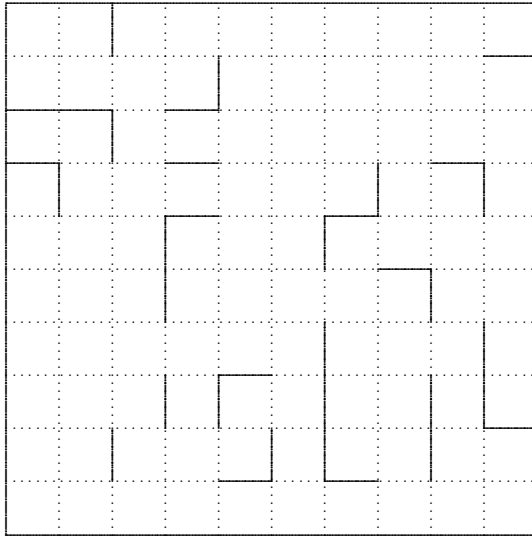
Problem 2



Problem 3

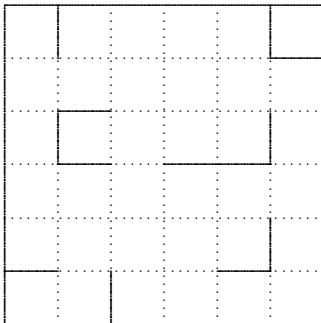


Problem 4

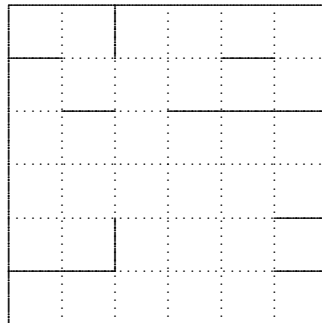


In Problems 5 to 15, two squares separated by a segment must contain identical numbers, but two identical numbers in adjacent squares need not be separated by a segment. Reconstruct the Tower Squares.

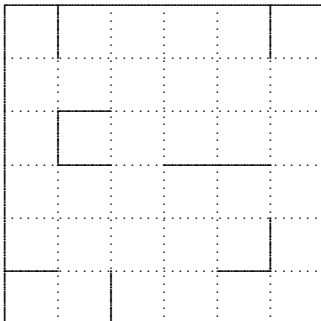
Problem 5



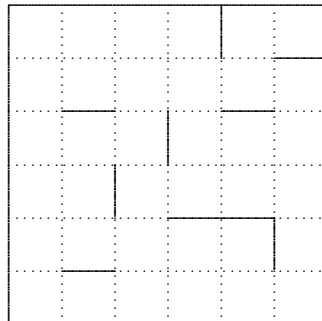
Problem 6



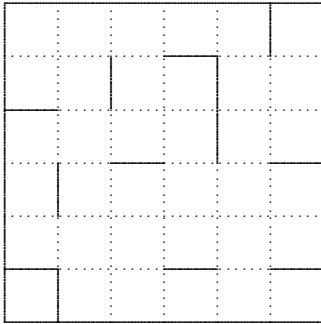
Problem 7



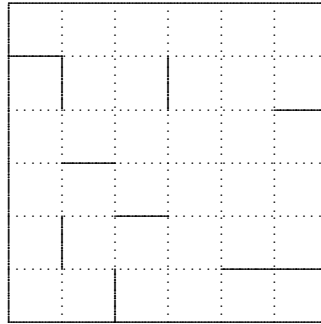
Problem 8



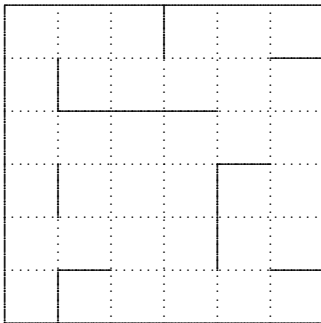
Problem 9



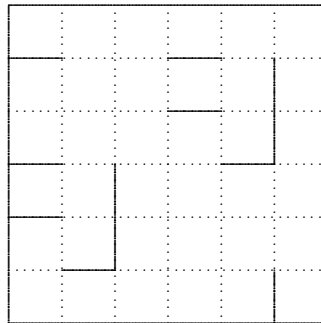
Problem 10



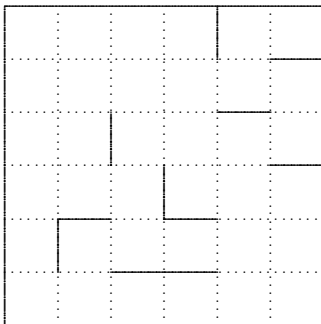
Problem 11



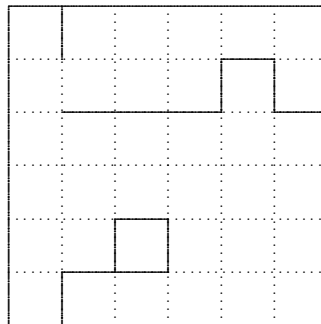
Problem 12



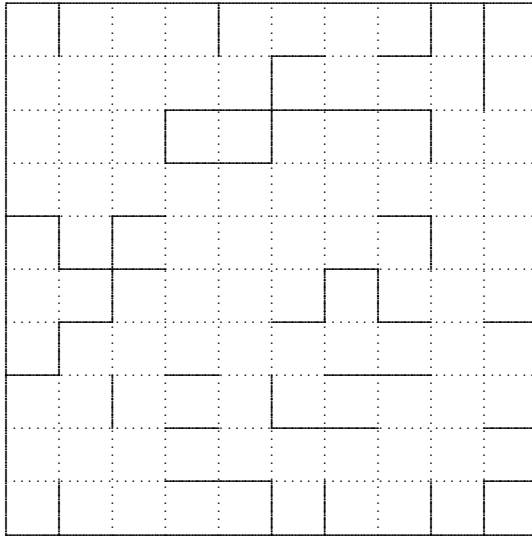
Problem 13



Problem 14

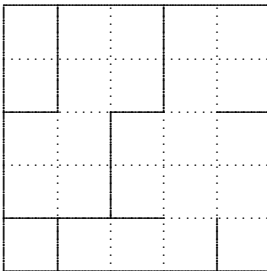


Problem 15

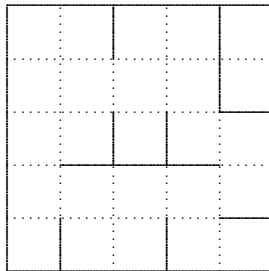


The 5×5 tables in Problems 16 to 18 are not quite Tower Squares. Each row and each column consists of 2 copies of 2 and 3 copies of 3. The rule is also different. Two squares separated by a segment must contain distinct numbers, but two distinct numbers in adjacent squares need not be separated by a segment. Reconstruct the squares.

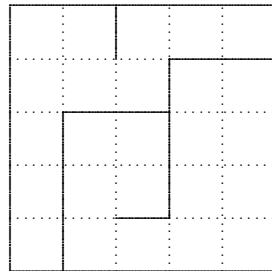
Problem 16



Problem 17



Problem 18



Answers

Problem 1

| | | | | | |
|---|---|---|---|---|---|
| 3 | 2 | 1 | 2 | 3 | 3 |
| 1 | 3 | 2 | 3 | 2 | 3 |
| 3 | 3 | 2 | 2 | 3 | 1 |
| 3 | 2 | 3 | 3 | 1 | 2 |
| 2 | 1 | 3 | 3 | 3 | 2 |
| 2 | 3 | 3 | 1 | 2 | 3 |

Problem 2

| | | | | | |
|---|---|---|---|---|---|
| 3 | 3 | 1 | 3 | 2 | 2 |
| 2 | 3 | 2 | 3 | 1 | 3 |
| 1 | 2 | 3 | 2 | 3 | 3 |
| 3 | 2 | 2 | 3 | 3 | 1 |
| 3 | 3 | 3 | 1 | 2 | 2 |
| 2 | 1 | 3 | 2 | 3 | 3 |

Problem 3

| | | | | | |
|---|---|---|---|---|---|
| 3 | 2 | 3 | 2 | 1 | 3 |
| 3 | 3 | 1 | 3 | 2 | 2 |
| 1 | 2 | 3 | 2 | 3 | 3 |
| 2 | 3 | 2 | 3 | 3 | 1 |
| 3 | 1 | 3 | 3 | 2 | 2 |
| 2 | 3 | 2 | 1 | 3 | 3 |

Problem 4

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 4 | 2 | 1 | 4 | 3 | 2 | 4 | 3 |
| 4 | 3 | 2 | 4 | 4 | 2 | 4 | 3 | 1 | 3 |
| 4 | 3 | 3 | 4 | 2 | 3 | 1 | 4 | 2 | 4 |
| 4 | 4 | 1 | 4 | 3 | 4 | 3 | 3 | 2 | 2 |
| 2 | 1 | 4 | 4 | 2 | 3 | 3 | 4 | 3 | 4 |
| 3 | 4 | 3 | 3 | 4 | 1 | 2 | 4 | 4 | 2 |
| 4 | 2 | 4 | 2 | 3 | 4 | 4 | 1 | 3 | 3 |
| 1 | 4 | 3 | 3 | 3 | 2 | 2 | 4 | 4 | 4 |
| 3 | 2 | 2 | 1 | 4 | 4 | 4 | 3 | 3 | 4 |
| 2 | 3 | 4 | 3 | 4 | 3 | 4 | 2 | 4 | 1 |

Problem 5

| | | | | | |
|---|---|---|---|---|---|
| 2 | 2 | 3 | 1 | 3 | 3 |
| 2 | 3 | 3 | 2 | 1 | 3 |
| 3 | 3 | 1 | 3 | 2 | 2 |
| 1 | 3 | 3 | 3 | 2 | 2 |
| 3 | 1 | 2 | 2 | 3 | 3 |
| 3 | 2 | 2 | 3 | 3 | 1 |

Problem 6

| | | | | | |
|---|---|---|---|---|---|
| 2 | 3 | 3 | 2 | 3 | 1 |
| 2 | 3 | 1 | 3 | 3 | 2 |
| 1 | 3 | 2 | 3 | 3 | 2 |
| 3 | 1 | 3 | 2 | 2 | 3 |
| 3 | 2 | 2 | 3 | 1 | 3 |
| 3 | 2 | 3 | 1 | 2 | 3 |

Problem 7

| | | | | | |
|---|---|---|---|---|---|
| 2 | 2 | 3 | 1 | 3 | 3 |
| 2 | 3 | 3 | 2 | 1 | 3 |
| 3 | 3 | 1 | 3 | 2 | 2 |
| 1 | 3 | 3 | 3 | 2 | 2 |
| 3 | 1 | 2 | 2 | 3 | 3 |
| 3 | 2 | 2 | 3 | 3 | 1 |

Problem 8

| | | | | | |
|---|---|---|---|---|---|
| 2 | 3 | 3 | 2 | 3 | 1 |
| 2 | 3 | 1 | 3 | 3 | 2 |
| 1 | 3 | 2 | 3 | 3 | 2 |
| 3 | 1 | 3 | 2 | 2 | 3 |
| 3 | 2 | 2 | 3 | 1 | 3 |
| 3 | 2 | 3 | 1 | 2 | 3 |

Problem 9

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2 | 3 | 3 |
| 3 | 3 | 3 | 2 | 2 | 1 |
| 3 | 1 | 2 | 3 | 3 | 2 |
| 3 | 3 | 2 | 1 | 3 | 2 |
| 2 | 3 | 1 | 3 | 2 | 3 |
| 2 | 2 | 3 | 3 | 1 | 3 |

Problem 10

| | | | | | |
|---|---|---|---|---|---|
| 3 | 1 | 2 | 2 | 3 | 3 |
| 3 | 3 | 2 | 2 | 1 | 3 |
| 2 | 2 | 1 | 3 | 3 | 3 |
| 2 | 2 | 3 | 3 | 3 | 1 |
| 3 | 3 | 3 | 1 | 2 | 2 |
| 1 | 3 | 3 | 3 | 2 | 2 |

Problem 11

| | | | | | |
|---|---|---|---|---|---|
| 3 | 1 | 2 | 2 | 3 | 3 |
| 2 | 2 | 3 | 3 | 1 | 3 |
| 1 | 2 | 3 | 3 | 2 | 3 |
| 3 | 3 | 3 | 2 | 2 | 1 |
| 2 | 3 | 1 | 3 | 3 | 2 |
| 3 | 3 | 2 | 1 | 3 | 2 |

Problem 12

| | | | | | |
|---|---|---|---|---|---|
| 2 | 3 | 3 | 3 | 1 | 2 |
| 2 | 1 | 2 | 3 | 3 | 3 |
| 3 | 3 | 1 | 3 | 2 | 2 |
| 3 | 3 | 3 | 2 | 2 | 1 |
| 3 | 2 | 2 | 1 | 3 | 3 |
| 1 | 2 | 3 | 2 | 3 | 3 |

Problem 13

| | | | | | |
|---|---|---|---|---|---|
| 3 | 1 | 3 | 2 | 2 | 3 |
| 3 | 2 | 1 | 2 | 3 | 3 |
| 2 | 3 | 3 | 1 | 3 | 2 |
| 2 | 3 | 3 | 3 | 1 | 2 |
| 3 | 3 | 2 | 3 | 2 | 1 |
| 1 | 2 | 2 | 3 | 3 | 3 |

Problem 14

| | | | | | |
|---|---|---|---|---|---|
| 3 | 3 | 1 | 2 | 3 | 2 |
| 1 | 2 | 2 | 3 | 3 | 3 |
| 3 | 2 | 2 | 3 | 1 | 3 |
| 2 | 1 | 3 | 3 | 2 | 1 |
| 2 | 3 | 3 | 3 | 2 | 1 |
| 3 | 3 | 3 | 1 | 2 | 2 |

Problem 15

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 3 | 2 | 2 | 4 | 1 | 4 | 4 | 4 |
| 2 | 1 | 4 | 3 | 4 | 4 | 2 | 4 | 3 | 3 |
| 3 | 2 | 3 | 3 | 4 | 4 | 2 | 4 | 4 | 1 |
| 4 | 3 | 4 | 3 | 4 | 1 | 4 | 2 | 2 | 3 |
| 4 | 4 | 4 | 1 | 3 | 4 | 3 | 2 | 2 | 3 |
| 1 | 4 | 4 | 2 | 4 | 3 | 3 | 3 | 4 | 2 |
| 4 | 4 | 2 | 4 | 1 | 3 | 4 | 3 | 3 | 2 |
| 4 | 3 | 3 | 4 | 2 | 2 | 4 | 3 | 1 | 4 |
| 3 | 4 | 2 | 4 | 3 | 2 | 4 | 1 | 3 | 4 |
| 2 | 2 | 1 | 4 | 3 | 3 | 3 | 4 | 4 | 4 |

Problem 16

| | | | | |
|---|---|---|---|---|
| 3 | 2 | 3 | 2 | 3 |
| 2 | 3 | 3 | 2 | 3 |
| 3 | 3 | 2 | 3 | 2 |
| 2 | 3 | 2 | 3 | 3 |
| 3 | 2 | 3 | 3 | 2 |

Problem 17

| | | | | |
|---|---|---|---|---|
| 3 | 2 | 3 | 2 | 3 |
| 2 | 3 | 3 | 3 | 2 |
| 3 | 2 | 3 | 2 | 3 |
| 3 | 3 | 2 | 3 | 2 |
| 2 | 3 | 2 | 3 | 3 |

Problem 18

| | | | | |
|---|---|---|---|---|
| 3 | 2 | 3 | 2 | 3 |
| 3 | 3 | 2 | 3 | 2 |
| 3 | 2 | 3 | 2 | 3 |
| 2 | 3 | 2 | 3 | 3 |
| 2 | 3 | 3 | 3 | 2 |