# Introduction to Part IV

Part III described how the American mathematical community evolved from a few scattered individuals in 1876 to a small but robust community boasting a professional society, five journals, and several graduate programs by 1900. That community crystallized in the ensuing period from 1900 to 1930, reaching the brink of international stature. These three decades featured a steady growth in terms of quantity and quality, as well as a consolidation of mathematical results. Hence the title of Part IV.

The period 1900–1930 is sometimes viewed as little more than a holding pattern in the history of mathematics in America, sandwiched between revolutionary changes in higher education from the preceding quarter century and the attendant rise in international acclaim resulting from émigré mathematicians escaping Nazi Germany. However, this chapter shows that major developments during these seemingly quiet decades foreshadowed the shape of the profession into a form recognizable today.

Two quantitative measures provide firm evidence of a crystallized mathematical community by 1930. Regarding organizational activity, AMS membership rose from 347 members in 1900 to 1926 members by 1930. Even though only a small percentage of both groups engaged in original research, the five-fold increase speaks of a community large enough to embrace various interests and specialties. Another measure is the development of, and products of, nascent graduate programs instituted at many universities throughout the US and, for the first time, Canada. Whereas in 1900 the US boasted but eight universities housing functioning graduate programs in mathematics (Chicago, Harvard, Yale, Johns Hopkins, Cornell, Columbia, Clark, and Bryn Mawr), by 1920 that list would include at least six more (the private universities Princeton and Pennsylvania, plus four land-grant universities, California at Berkeley, Illinois, Michigan, and Wisconsin.) These fourteen institutions accounted for nearly 90% of the 406 doctoral degrees in mathematics awarded in the US between 1900 and 1920. Moreover, by 1930 the departments at MIT, Brown, and Stanford were rivaling the older established programs in quality and quantity.

Yet perhaps the best measures of a crystallized American mathematical community, though metrics are harder to gauge than mere quantification, are the quality of faculty productivity and the success of their graduates. Here too, overwhelming evidence points to a community that had not only grown in numbers but which had broadened its interests, expanded its expertise, and gained international recognition.

Whereas the first five presidents of the NYMS/AMS, with terms falling between 1888 and 1900, were applied mathematicians, beginning with E.H. Moore in 1900, every AMS president emphasized pure mathematics, including even the astronomer Ernest Brown (AMS president, 1915–1916). In 1912 the tenth president, Maxime Bôcher, became the first American pure mathematician invited to deliver an address at an International Congress of Mathematicians. American activity at these quadrennial events is traced as a gauge for measuring increasing international recognition.

Rather than viewing the period of growth and consolidation as one monolithic entity, it is helpful to impose a structure by dividing it into three subperiods of different durations. Chapter 8 covers the pre-World War I years (1900–1914), when leadership was furnished partly by J.J. Sylvester's students, but more importantly by students supervised by Felix Klein and David Hilbert at Göttingen, Sophus Lie at Leipzig, and, to an increasing degree throughout the period, E.H. Moore at Chicago.

Chapter 9 covers the World War I years, defined here as 1914–1920, a time when war work was carried out by mathematicians in the US—more than was generally known until recently. Before treating this aspect of the six-year period, the chapter begins with the founding of the second professional organization of mathematicians, the Mathematical Association of America (MAA). Devoted to collegiate mathematics, the MAA quickly became a vital force in the American mathematics community, its sectional meetings embracing large numbers of teachers who would not have attended AMS gatherings. After treating various ways in which US mathematicians participated in WWI, the chapter describes some advances in cryptology, as well as another American specialty, topology (initially called *analysis situs*). The final section discusses the first wave of Chinese students who came to the US to seek doctorates in mathematics.

Chapters 10 and 11 are devoted to the 1920s, when the American mathematical community experienced a burgeoning number of contributors and notable advances in a broader range of specialized fields. Chapter 10 describes how some figures impacted the leading mathematics departments—Birkhoff at Harvard, Veblen and Lefschetz at Princeton, and Wiener at MIT. Chicago, however, stagnated in the 1920s without such vigorous leadership. The faculties at Yale (Ore and Hille) and Brown (Richardson) were also upgraded considerably.

Chapter 11 discusses improvements in the growing community and their subsequent advances. It begins with accounts of the first three African-American pioneers to earn doctorates in mathematics. Part of that story took place in Canada, whose entry into research mathematics is discussed next. Similar to the emergence in the US that was led by one individual, J.J. Sylvester, developments in the 1920s in Canada were spearheaded by one person, John Charles Fields. First, Fields played a part in convincing the AMS to hold an annual meeting in Toronto (in 1921). Three years later, he was the prime mover in hosting an International Congress of Mathematicians, also in Toronto. One of the most internationally connected mathematicians in the world at the time, Fields spent the last few years of his life circa 1930 in pursuit of an international award to smooth animosities resulting from World War I—today called the Fields Medal and viewed as the most prestigious award in mathematics.

The remaining part of Chapter 11 addresses other developments that took place in the US in the 1920s. It begins with an account of post-doctoral fellowships in mathematics. Next, the chapter describes three changes that took place within the AMS; it concludes with a description of the first mathematics buildings that housed mathematicians in private offices, along with seminar rooms and a departmental library.

The Transition 1930 section focuses on the Albert–Hasse affair that, once again, contrasts faculties in America with those in Germany. It serves as a bridge between two time periods, and it motivates issues that characterized American mathematics in the 1930s, which will be documented in Part V.

# 8

# Establishment: 1900–1914

Curricula at American colleges in the first part of the twentieth century offered an impressive range of courses. The historian of mathematics David E. Smith cited a study comparing America with leading European countries at the time [**504**, p. 76]:

> A recent investigation (1903) showed that 67 students in ten American institutions were taking courses in the theory of functions, 11 in the theory of elliptic functions, 94 in projective geometry, 26 in the theory of invariants, 45 in the theory of groups, and 46 in the modern advanced theory of equations, courses which only a few years ago were rarely given in this country. … The interest at present manifested by American scholars is illustrated by the fact that only four countries (Germany, Russia, Austria, and France) had more representatives than the United States, among the 336 regular members at the third international mathematical congress at Heidelberg in 1904.

The most important developments during 1900–1914 occurred at one institution, the University of Chicago, so this chapter begins with three figures supervised there by E.H. Moore—Oswald Veblen, R.L. Moore, and George Birkhoff—who, along with fellow Moore student Leonard Eugene Dickson,[1] were instrumental in the development of mathematics in America in myriad ways through the 1960s.

Oswald Veblen was set to launch a career that would make him one of the leading mathematicians in the world. Because he spent the bulk of his career at Princeton, a discussion of students at Chicago naturally extends to faculty members at Princeton. E.H. Moore at Chicago and Henry Fine at Princeton were two of the four leading figures in the American mathematical research community during the first fourteen years of the twentieth century. Fine is more remembered for administrative work, while Moore, along with Harvard's William Osgood, Maxine Bôcher, and George Birkhoff, became major figures in mathematical research. The ages for these figures in 1900 ranged from 33 to 42; Veblen and Birkhoff were even younger. Each was in the prime of his productive life.

[1] Dickson, an 1896 PhD student of Moore, was profiled in Volume 1.

## 8.1 Moore Mob, II

Up to 1900 many aspiring Americans felt compelled to travel abroad to complete their studies in mathematics (as doctoral students) or to enhance their learning (as visiting professors). The number of such mathematicians swelled during the 1890s but dwindled to a trickle during the first decade of the twentieth century. Germany remained the leading country for mathematics in the world, so many Americans continued to attend lectures at its various universities to take advantage of top-notch faculties in all fields. Others, however, preferred England, France, or Italy for particular topics or to study under specific individuals. Yet, after 1900 the bulk of aspiring mathematicians in the American community stayed at home, not only to enroll in doctoral programs in fledgling departments, but also to keep abreast of current events on research topics by attending AMS meetings, thereby interacting with others who shared mutual interests.

The University of Chicago assumed the mantle of leadership, with E.H. Moore, Oskar Bolza, and Heinrich Maschke stitching together their own research agendas within the now-thriving graduate program. Moore supervised the work of one outstanding student in 1896, Leonard Dickson, just four years after Chicago opened its doors, while Bolza produced another, Gilbert Bliss, in 1900. Dickson and Bliss with Veblen, R.L. Moore, and Birkhoff provided substantial direction and leadership to the American mathematical community throughout the first half of the twentieth century.

Oswald Veblen (1880–1960) became one of the most influential mathematicians nationally in the first half of the twentieth century, along with George Birkhoff.[2] Veblen's paternal grandparents had emigrated from Norway to Wisconsin but then settled into a farming community in Minnesota. His parents had been raised on nearby Norwegian-American farms. One of Veblen's uncles was the famous economist and social theorist, Thorstein Veblen. His father, Andrew, taught mathematics, physics, and English at what is now the University of Iowa. Andrew Veblen enrolled at Johns Hopkins (1881–1883), but it is not known whether he pursued mathematics there. After that, Andrew returned to Iowa. His son Oswald graduated from Iowa at age eighteen, and then spent a year running his ailing father's physics labs. A year later, Oswald Veblen enrolled at the University of Chicago, earning a second bachelor's degree in 1900 while coming under the influence of department head E.H. Moore. Finding the mathematics department at Chicago to be a perfect fit, Veblen continued directly into graduate school, even without an offer of financial aid. However, another student with a fellowship withdrew from the university during the first quarter, whereupon he was awarded the stipend.

Veblen received his PhD in 1903 with a dissertation titled simply *A system of axioms for geometry.* The inspiration for this material came from a seminar that E.H. Moore offered in the fall quarter of 1901 examining David Hilbert's work on the foundations of geometry. Hilbert had devised a scheme of 20 axioms for Euclidean geometry based on the undefined terms of point, line, and plane. Moore, correctly as it turned out, questioned the independence of these axioms [**378**]. In his dissertation, Veblen proposed a system of twelve axioms and proved that they were independent of one another. It was published the following year in the *Transactions of the AMS* and was cited in numerous subsequent works.

[2]The primary sources on Veblen are [**372**] and [**37**].

Figure 8.1. Oswald Veblen

Veblen spent the next two years on the Chicago faculty (1903–1905), where he had five colleagues: his mentor E.H. Moore, two professors Moore had hired (Oskar Bolza and Heinrich Maschke), and two of Moore's former students (Herbert Slaught and Leonard Dickson). Jumping into this hotbed of research activity in the fall of 1903 was a new graduate student, R.L. Moore, who came with the especially strong recommendation of George Halsted. The younger Moore made an immediate impact, forming a trio with Veblen and fellow graduate student Nels Lennes; R.L. Moore earned his PhD degree in 1905 at age 23 and Lennes received his two years later. These three friends worked symbiotically from the start; for instance, in 1905 Veblen and Lennes completed the book *Introduction to Infinitesimal Analysis, Functions of One Real Variable*, which was published in 1907 and garnered sufficient demand for a reprinting in 1935. Veblen, in fact, served as an informal co-supervisor of R.L. Moore's dissertation.

The other towering figure to emerge at Chicago at about the same time, George Birkhoff, followed this same pattern by earning his PhD in 1907 at age 23. But by the time Birkhoff began work on his dissertation supervised by E.H. Moore, his advisor's interest had turned along a new track, so although Birkhoff was from the same academic generation, his specialty was along entirely different lines.

Like Oswald Veblen, George David Birkhoff (1884–1944) was one of the most highly regarded American research mathematicians during the first four decades of the twentieth century.[3] His climb to this lofty status did not occur by chance; rather it received a strong impetus from one of the very best educational systems available in the country at that time. Of Dutch descent, his parents lived in Chicago, where his father, a physician, provided his family with the best possible education. His son George attended the prestigious Lewis Institute (which ultimately became Illinois Institute of Technology), where at age fifteen he solved a geometry problem posed in the Problems Section of the *American Mathematical Monthly* . He entered the University of Chicago as a junior in 1902 but transferred to Harvard for his senior year with the blessing of

[3]For more details on the life and work of Birkhoff, see [**389**].

Chicago's head, E.H. Moore. The brilliant Birkhoff stayed two years at Harvard, the first as an undergraduate and the second as a graduate student. During this time, he was highly influenced by analysis courses taken from Maxime Bôcher, whom he later credited "for his suggestions, for his remarkable critical insight, and his unfailing interest in the often-crude mathematical ideas which I presented" [**78**, p. 275]. Birkhoff also published a joint paper with Harry Vandiver on integral divisors of $a^n - b^n$ that contained a number-theoretic result ultimately pivotal in Wedderburn's proof of his eponymous theorem the next year.

Consequently, by the time Birkhoff enrolled in the graduate program at Chicago in the fall of 1905, he had already completed requirements for both the bachelor's and master's degrees from Harvard. Just one year later, he published two articles, one of which appeared in the *Transactions of the AMS*. Birkhoff was awarded his Chicago PhD in 1907 for the dissertation *Asymptotic properties of certain ordinary differential equations with applications to boundary value and expansion problems.* This work shows the influence of both Bôcher (on expansion theorems) and his official supervisor, E.H. Moore (on integral equations). Birkhoff published the dissertation in two parts in the *Transactions of the AMS*. After receiving his doctorate, Birkhoff spent two years at the University of Wisconsin, where another new faculty member was Edward Van Vleck, a former student of Felix Klein. The remainder of Birkhoff's career is traced below.

Figure 8.2. George David Birkhoff

E.H. Moore, as related in Volume 1, had enticed Leonard Dickson back to Chicago in 1900. This appointment paid dividends at once as the Texan set about a research program in algebra and number theory that would last a lifetime. Dickson joined the effort at producing doctorates within a year of his hiring. After obtaining his doctorate under E.H. Moore in 1896, Dickson spent a year in Europe before accepting an instructorship at the University of California at Berkeley. Here he met Thomas Milton Putnam (1875–1942), a fellow in mathematics (just one year younger than Dickson) who had just received his BS degree. Putnam became the first of many students that Dickson mentored, earning a master's degree in 1899 before moving to Texas with Dickson to

Table 8.1. Chicago PhD recipients 1900–1914

| **Student** | **Year** | **Advisor** |
|---|---|---|
| T.M. Putnam | 1901 | Dickson |
| Oswald Veblen | 1903 | Moore |
| R.L. Moore | 1905 | Moore |
| W.R. Longley | 1906 | Unknown |
| G.D. Birkhoff | 1907 | Moore |
| N.J. Lennes | 1907 | Moore |
| F.W. Owens | 1907 | Moore |
| Mary Sinclair | 1908 | Bolza |
| Arnold Dresden | 1909 | Bolza |
| T.H. Hildebrandt | 1910 | Moore |
| A.D. Pitcher | 1910 | Moore |
| Anna Johnson Pell Wheeler | 1910 | Moore |
| E.W. Chittenden | 1912 | Moore |
| C.T. Sullivan | 1912 | Wilczynski |

continue his studies. When Dickson left Austin for Chicago the next year, Putnam traveled with him once again.

Consequently, when Putnam completed Chicago's PhD requirements in 1901 based largely on work done before arriving at the university, he became the first of 67 doctorates that the indefatigable Dickson produced over the next 37 years. Upon receiving his degree, Putnam accepted a position at his *alma mater*, Berkeley, where he spent the rest of his life. During his early years, he wrote several articles for the *American Mathematical Monthly*, but beginning in 1914 he accepted administrative positions that curtailed his output. In 1923, however, Putnam produced his best-known book, *Mathematical Theory of Finance*. He served the AMS in two different ways, as secretary of the San Francisco Section during 1911–1912 and as associate secretary of the national AMS from 1930 to 1940. He is not related to the family that donated funds for today's famous undergraduate Putnam Competition.

We now turn our attention to several notable figures who received doctorates from Chicago during 1900–1914. Perhaps less prominent than the towering figures Veblen, Moore, and Birkhoff, who soared over the American landscape and ultimately became international leaders, each of these individuals contributed to the American community in vital ways. Table 8.1 lists fourteen notable graduates along with the individual year of graduation and supervisor (except Longley).

The first three figures in Table 8.1 were introduced already. The fourth, W.R. Longley, became a mainstay on the faculty at Yale, where one student remembered his manner of developing creativity in his students and then mentoring them into the process of presenting their results in front of an audience [**161**, p. 93]:

> W.R. Longley—a pleasant, dignified professor—conducted his class in a somewhat formal manner. In the one course I took with him, near the beginning he assigned to each member of the class a specific topic that we were to digest on our own and then make a presentation to the class near the end of the term. I do not remember this procedure being followed in any other course I took at Yale.

Longley was one of three Yale authors of the textbook *Elements of the Differential and Integral Calculus*, which has been described as "the standard calculus text, the book against which others were measured in the United States for nearly five decades" [**465**, p. 99].

Nels Johann Lennes graduated in 1907 and, after short-term appointments at MIT and Columbia, moved to the wilderness of Missoula, Montana and assumed a professorship at the University of Montana. Lennes became a leading topologist known mainly for his definition of connected sets. He also was the first to prove that a polygon can be triangulated [**373**]. Two who received PhDs in 1910 were Theophil Henry Hildebrandt (1888–1980) and Arthur Dunn Pitcher (1880–1924). Hildebrandt joined the faculty at the University of Michigan in 1909 and chaired it during a critical period in its history, from 1934 until his retirement in 1957.

Figure 8.3. Theophil Hildebrandt (center)

Pitcher was the father of mathematician Arthur Everett Pitcher who was Secretary of the AMS 1966-1988. Another respected figure in Table 8.1 is Edward Wilson Chittenden (1885–1977), who became a leader in topology at Iowa State University. The first woman to receive a doctorate at Chicago, Mary Emily Sinclair (1878–1955), completed her dissertation on the calculus of variations under Oskar Bolza in 1908. She taught at Oberlin, the first coeducational college in the US, from 1907 up to her retirement in 1944, succeeding William Cairns (a student of David Hilbert) as chair of the department (1939–1944). Sinclair's life was unusual for her time—not only did she have a decades-long career in academia but also she raised two adopted children, a boy and a girl, despite never marrying.[4]

[4]Claudia Goldin has outlined the progression of women in the workplace in the twentieth century in her articles "Career and Family: College Women Look to the Past" and "The Quiet Revolution that Transformed Women's Employment, Education and Family." Goldin argued that in the early twentieth century, women had to choose between family or career. By mid-century, women often had marriage and children first, then pursued a career. In the middle 1960s and 1970s, women chose career and then family serially. Beginning in the 1980s women chose career and family concurrently. Goldin referred to the final stage of development, where women finally came to pursue career and family as a "quiet revolution" that necessarily followed the earlier "evolutionary" phases [**220**, **221**].

Four of the twenty-four females who earned PhDs in math in the US during 1900–1910 earned those degrees at Chicago. Helen Barten Brewster Owens (1881–1968) was not one of them; she studied at Chicago but received her PhD in 1910 at Cornell under Virgil Snyder. Her husband, Frederick William Owens (1880–1961), a 1907 graduate under Moore, taught initially at Cornell before moving to Penn State. Since the Owens's dissertations were directed by Snyder and Bolza, former students of Felix Klein, the married couple forms another connection between Göttingen and Chicago. They left perhaps their biggest imprint at Penn State from the time they moved there from Ithaca in 1926. Throughout their careers, Frederick and Helen Owens were confronted by the "two-body problem," whereby couples attempt to secure geographically close positions, with a clear preference for employment at the same university. Christine Ladd and Fabian Franklin were the first mathematical married couple to face this challenge in the US. They never succeeded. Nor did our next intriguing pair.

Anna Johnson Pell Wheeler (1883–1966) studied at Göttingen under David Hilbert but left without earning a degree after a disagreement with her supervisor. Apparently, she felt that her dissertation was complete; he disagreed. While in Göttingen she married Alexander Pell (1857–1921), whose life reads like that of a character in a Dostoevsky novel. Born Sergei Dagaev in Moscow, he became involved in a radical revolutionary group that assassinated Czar Alexander II, but then he switched sides and informed on the group for the secret police. After which he turned face again, killing the head of the secret police for his radical group. To escape, he immigrated to England, and in 1886 sailed to the US. He and his Russian wife moved around the country, taking menial jobs; he adopted the name Alexander Pell when he became a naturalized citizen in 1891. Four years later the Pells moved to Baltimore, where he enrolled in the graduate program at Johns Hopkins, earning his PhD in 1897. Upon graduation, Pell took up a professorship at the newly founded University of South Dakota (USD), where Anna Johnson was an outstanding student. She received a bachelor's degree in 1903 under his tutelage, and a master's degree the next year at the University of Iowa. She obtained a second master's in 1905 from Radcliffe College, the coordinate college whose institutional affiliation with Harvard allowed her to take classes with Bôcher and Osgood. That fall, she won a fellowship to study in Göttingen. Two years later, Alexander Pell traveled to the quaint German burg to marry her (three years after his first wife had died), whereupon the newlyweds returned to the US as faculty members at USD. Alexander Pell resigned his position the following year to teach at the Armour Institute of Engineering in Chicago, while Anna Pell returned to Göttingen to complete her dissertation. But Hilbert refused to sanction her work, so she headed back to Chicago to enroll in E.H. Moore's program. In 1910, Moore accepted the dissertation which was about a sort of proto-functional analysis, a topic just being developed at the time and an interest Moore shared with Hilbert.

Anna Pell cared for her husband after his stroke the next year. She initially held positions at Mount Holyoke College (1911–1918) and then Bryn Mawr College (1918–1948), where she directed eight doctoral students. Her first, Margaret Buchanan Cole (1885–1959), wrote the dissertation *Systems of two linear integral equations with two parameters and symmetrizable kernels.* Upon graduation in 1922, Buchanan taught at West Virginia 1922–1929 until her marriage forced her resignation. She returned to the university 1938–1955. Wheeler's final doctoral student, Dorothy Maharam Stone (1917–2014), a 1937 graduate of Carnegie Tech, received her Bryn Mawr PhD in 1940

Figure 8.4. Anna Johnson Pell Wheeler

for the dissertation, *On measure in abstract sets.* She spent the next year on a postdoctoral fellowship at the Institute for Advanced Study where she met, and then married, the British mathematician Arthur Harold Stone (1916–2000). Both Stone and Maharam Stone were on the faculty at the University of Rochester. Maharam Stone was the only one of Wheeler's eight students to supervise three of her own students (at Rochester 1975–1984). The Stone's two children, David and Ellen, both became mathematicians as well.

Anna Pell married Bryn Mawr professor of classics Arthur W. Wheeler in 1925, four years after the death of Alexander Pell. As head of the mathematics department at Bryn Mawr, Anna Pell Wheeler was partly responsible for bringing Emmy Noether to campus in 1933. Wheeler was one of the most accomplished and recognized female mathematicians in the first half of the twentieth century: she gave the first invited address by a woman at an AMS meeting, she delivered the prestigious Colloquium Lectures in 1927, and was an editor of *The Annals of Mathematics* from 1927 to 1945. Most of her mathematical work was in analysis, broadly defined, including differential equations, integral equations, and functional analysis.

The life of Arnold Dresden (1882–1954) was not as dramatic as that of Anna Pell Wheeler's, but it had its share of drama. Dresden had been a student at the University of Amsterdam when he received a message from a friend in need of help. So, against his parents' wishes, he used money set aside for tuition to pay for a transatlantic voyage to New York and thence to Chicago, where he arrived on his twenty-first birthday. After working at several menial jobs, he enrolled in E.H. Moore's department. Dresden received his doctorate in 1909, the last student of Oskar Bolza. Dresden stayed at the University of Wisconsin through 1927 and became a prominent figure in the AMS while there. However, his primary focus had changed from the calculus of variations to undergraduate education, resulting in a move to initiate an honors program at Swarthmore College in suburban Philadelphia. That program became a model for elite, small colleges around the country and, in the main, is still operational at Swarthmore today. Swarthmore is very close to Bryn Mawr, so Anna Pell Wheeler and Arnold Dresden enjoyed many social and professional contacts throughout the years, joining with John Kline of the University of Pennsylvania to form a symbiotic relationship that benefited all three mathematicians and all three institutions. Kline was a PhD student of R. L. Moore and thus an academic grandson of E. H. Moore, he worked in point-set topology and from 1941–1950 served as secretary of the AMS.

The remaining figure in Table 8.1 is Charles Thompson Sullivan (d. 1948), who was born and educated in New Glasgow (NS) [**207**, p. 192]. He received a bachelor's degree from Dalhousie University (Halifax, NS) and a master's degree from McGill University (Montreal, ON) before joining the McGill faculty in 1908, four years before obtaining his doctorate. This suggests that, like many aspiring mathematicians at the time, he took summer courses at the University of Chicago. He earned his PhD in 1912 for the dissertation, *Properties of surfaces whose asymptotic lines belong to linear complexes,* directed by Ernest Wilczynski. Sullivan wrote over thirty articles and directed the honors program during his 39-year tenure at McGill. After Lloyd Williams joined the mathematics department in 1924, Sullivan and Williams got together every year and worked through a book or a series of papers, which was as close to a faculty seminar as McGill would have for another two decades. Sullivan retired from McGill in 1946.

In addition to the fourteen figures in Table 8.1, Chicago graduated about three dozen other doctorates in mathematics over the period 1900–1914. To the best of our knowledge, the only one born abroad was Nels Lennes, yet even he immigrated to the US at age sixteen. The production of nearly four PhDs per year at Chicago alone reinforces not only the switch from foreign to North American universities for the ultimate degree but also attests to the rapid rise in numbers of Americans seeking the highest degree.

## 8.2 Bringing Hilbert to America

Mathematicians worldwide celebrated the start of the new century by holding the second International Congress of Mathematicians (ICM) in Paris, almost nine months later than the general population popped bottles of champagne. This ICM, held in August 1900, is mainly remembered today as the venue where David Hilbert presented his famous list of problems. Initially, Hilbert vacillated on a topic during the weeks leading up to his presentation in the hot and sultry lecture hall in the Sorbonne. Beforehand, he vetted his idea of the future direction of mathematics with his Göttingen colleague Hermann Minkowski, who responded, "Most alluring would be the attempt at a look into the future and a listing of the problems on which mathematicians should try themselves during the coming century. With such a subject you could have people talking about your lecture decades later" [**438**, p. 69]. Minkowski was shy of the mark—mathematicians are still attempting to solve some of these problems a *century* later.

Yet Hilbert's famous presentation was *not* one of the plenary lectures. The four principal speakers comprised an international mix of Moritz Cantor of Heidelberg, Gösta Mittag-Leffler of Stockholm, Vito Volterra of Rome, and Henri Poincaré of Paris. Admittedly this was a star-studded cast, but Hilbert was not a principal speaker, even though his achievements to that point were superior to all four lecturers except perhaps Poincaré. Instead, Hilbert's address, *Mathematische Probleme*, was delivered in German at a special joint meeting of two sections, History of Mathematics and Pedagogy of Mathematics. Yet, by the time the *Comptes Rendus* (Proceedings) of the Congress were published, the historic presentation was placed among the plenary lectures "due to its great importance" [our translation]. Indeed, the printed version appeared in French with the title *Problèmes futures des mathématiques.*

Hilbert's problems have played an important role in the development of mathematics since he first enunciated them, but it is not our place here to discuss whether this is due to his ability to single out the most important concepts or to his international reputation. After all, what aspiring mathematician, or even seasoned veteran, would not want to establish, or to cement, a lifetime reputation by solving a problem posed by someone already regarded as one of the top mathematicians in the world, arguably second only to the Frenchman Poincaré? Hilbert played an important role in the development of mathematics in North America. Here, we describe how the crystallizing community reacted quickly with reports on his address and its translation into English. The section "Transition 1900" has already described the small colony of American students who earned doctorates under him in Göttingen.

The emerging community of research mathematicians in America quickly realized the importance of Hilbert's lecture; three made use of relatively new outlets to spread word of it to the English-speaking world soon after Hilbert finished enumerating his seminal list of 23 problems.[5] In fact, the Americans were amazingly quick with reports in the two journals that served as beacons for that community. First, the August-September 1900 issue of the *American Mathematical Monthly* contained a two-page report on the ICM by George Halsted, appearing almost simultaneously with the meeting. After describing the president of the ICM, Henri Poincaré, as "the greatest of living mathematicians," Halsted discussed attendance at the event [**243**, p. 188]. Beforehand it was expected that over 1000 mathematicians would attend, but a disappointing 229 appeared. Halsted singled out three Americans who attended: Charlotte Angas Scott, Irving Stringham, and Johannes Hagen. Scott was the department head at Bryn Mawr College, and Stringham, a PhD student of J.J. Sylvester at Johns Hopkins who also studied two years in Leipzig under Felix Klein, was at Berkeley.

The Austrian-born Jesuit priest Johannes Georg Hagen (1847–1930) had come to the United States to teach in 1880. Known in the US as John George Hagen, within two years of arrival, he published a short note in the *American Journal of Mathematics* on the division of one power series by another. In 1888 he was appointed director of the astronomical observatory at Georgetown University in Washington, DC, where he remained until 1905. During this time, he published one of the first bibliographies of the works of the famous Swiss mathematician Leonhard Euler called *Index operum Leonardi Euleri* (1896). Since it is written in Latin, the author is listed as Ioanne G. Hagen, S.J. [**586**, p. 256]. In 1906 Father Hagen was called to Rome to head up the Vatican Observatory. He never returned to America, but he was remembered favorably by its mathematical community. An *American Mathematical Monthly* note from

[5] Rüdiger Thiele found evidence in one of Hilbert's unpublished notebooks that Hilbert had contemplated adding another problem—on logic—that he ultimately rejected [**526**].

1922 announcing one of Hagen's volumes stated, "The older members of the American Mathematical Society will recall with pleasure Father Hagen's presence at their meetings" [**28**, p. 172].

The volume mentioned in the *Monthly* note was the third in Hagen's four-volume series, *Synopsis der höheren Mathematik* (*Synopsis of Higher Learning*), which was published in Berlin between 1891 and 1930. In a review of the first volume (including the prospectus for the remaining volumes) from the *Annals* within a year of its publication, William Smith of the University of Missouri expressed a very serious frustration faced by mathematicians in the early 1890s. "One very grave obstacle to the successful prosecution in the United States of original research in mathematics has been the practical inaccessibility of its literature ... In four stately volumes [Prof. Hagen] seeks to summarize the results of as many centuries of investigation" [**511**, p. 84]. But not all reviewers were as welcoming. Although Illinois group theorist George Miller listed the three volumes that had been published by 1912 on his brief list of "useful mathematical books beyond elementary calculus," he grumbled about the language: "While the number of people who use the English language is very much larger than the number of those who use German ... and with the growth of scientific interest among English speaking nations, it would appear that the English scientific works should soon command the most extensive market" [**367**, p. 67].

While Halsted's two-page brief on the ICM was printed seemingly simultaneously with the meeting, a more detailed, 33-page account was submitted by Charlotte Angas Scott to the *Bulletin of the AMS* in October that year and printed in the November issue [**477**]. Scott reported that E.H. Moore was absent from the meeting even though he was a member of the executive board for the ICM (as a vice president). She stated that seventeen mathematicians attended from the US but could recall only fifteen names, as no attendance list was ever distributed. The Americans were Robert Allardice (Stanford), E.W. Brown (Haverford College), Leonard Dickson (Chicago), A.M. Ely (Vassar), Johannes Hagen (Georgetown), George Halsted (Texas), Harris Hancock (Cincinnati), James Harkness (Bryn Mawr), Herbert Keppel (Northwestern), Edgar Lovett (Princeton), Alexander Macfarlane (Ontario, Canada), Alexander Pell (South Dakota), Charlotte Scott (Bryn Mawr), Irving Stringham (Berkeley), and A.G. Webster (Clark). This list shows that American mathematicians from across the US attended the ICM, with three of the fifteen from women's colleges. Scott also reported that one Canadian mathematician was in attendance, but she supplied no name; she did not regard Macfarlane as Canadian even though the Scottish-born mathematical physicist had retired to Canada by then. Scott also stated that six Americans presented papers, but her report identifies only Lovett, Macfarlane, and Stringham.

If Scott's views are typical of those who attended the ICM, mathematicians were not as happy about meeting in such a huge city as opposed to Zurich three years earlier. She groused [**477**, p. 75]:

> The arrangements excited a good deal of criticism. ... There is no doubt that a smaller town lends itself best to such a gathering.

But her complaints did not stop there—she was equally unhappy with the oratorical abilities of many speakers (as opposed to those of David Hilbert, Gösta Mittag-Leffler, Vito Volterra, and Henri Poincaré). She protested that even though some speakers did not adhere to time limitations of either twenty minutes or one hour, they still seemed unable to isolate the *raison d'être* for their presentations: "If the author regards

Figure 8.5. Alexander Macfarlane

all details as equally important, his auditors will regard all as equally unimportant" [**477**, p. 77]. Her summary pulled no punches: "One thing very forcibly impressed on the listener is that the presentation of papers is usually shockingly bad. … It would be invidious and impertinent to mention names; the special sinners sit in both high and low places" [**477**, p. 77].

On the other hand, the Bryn Mawr professor proudly pointed out contributions by three Americans [**477**, p. 67]: "In Section III papers were presented by MM. [messieurs] Lovett, 'On contact transformations between the essential elements of space'; Macfarlane, 'Applications of space analysis to curvilinear coordinates'; Stringham, 'Orthogonal transformations in elliptic or in hyperbolic space."' Edgar Lovett was a PhD student of Sophus Lie who was instrumental in elevating Rice Institute to national prominence as its first president. The Scottish-born Alexander Macfarlane (1851–1913) came to America as professor of physics at the University of Texas in 1885. After holding that position for nine years, he taught electrical engineering at Lehigh University for two years before retiring to Canada in 1898. Macfarlane was very active in the American mathematical community, attending several AMS meetings as well as the Chicago Congress of 1893. He is mainly known today for developing "space analysis" (which presaged the velocity geometry of modern space-time theory) and for his book *Lectures on Ten British Mathematicians*. This book, based on lectures delivered at Lehigh in 1901, was published posthumously in 1916 at the urging of his wife, the former Helen Swearingen, whose sister Margaret was married to Macfarlane's Texas colleague George Halsted.

The ICM report by Charlotte Angas Scott provided a brief analysis of a few of Hilbert's problems but added [**477**, p. 68]:

> These are but a few of the problems that M. Hilbert mentioned, and these were a selection from a much longer list for which he referred to an article about to appear, … a rather desultory discussion … followed.

It took another two years before a translation of Hilbert's lecture in English was published in the AMS *Bulletin of the AMS* under the title "Mathematical problems." A footnote on the title page reveals that the paper was [**401**, p. 437]:

> Translated for the *Bulletin* with the author's permission by Dr. Mary Winston Newson. The original appeared in the *Göttingen Nachrichten* 1900, pp. 253–297 and in the *Archiv der Mathematik und Physik* **1** (1901), pp. 44–63 and 213–237.

Apparently, Winston-Newson based her translation on these two sources because she did not attend the ICM. Fittingly, her knowledge of German and flair for English are apparent already in her famous translation of Hilbert's introduction [**401**, p. 437]:

> Who among us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? … For the close of a great epoch not only invites us to look back into the past but also directs our thoughts to the unknown future.

Having married in 1900, Mary Winston was known as Mrs. Newson when her translation appeared in 1902. Although nepotism rules forced her to resign her position at Kansas State and she had not published any papers after her dissertation, she surely had kept abreast of the field, as her Hilbert translation is laced with footnotes demonstrating deep knowledge across a broad spectrum of mathematical specialties. For instance, after translating the material surrounding Problem 3, she wrote, "Since this was written, Herr Dehn succeeded in proving this impossibility. See his note … [from] 1900 and a paper soon to appear in the *Math. Annalen*" [**401**, p. 449]. Regarding another problem, she added, "Cf. an article by Herr von Koch soon to appear in *Math. Annalen*" [**401**, p. 457]. And when Hilbert praised a book by A. Kneser on the calculus of variations, she noted that the work "applies Weierstrass's theory also to the inquiry for the extreme of such quantities as are defined by differential equations" [**401**, p. 472]. Clearly, Winston-Newson had not only kept abreast of the literature, but she was also aware of unpublished material from the German school as well.

The German connection seen here was not directly evident in an international event that took place four years after the 1900 ICM in Paris. The next two sections recall that Congress as well as the AMS meeting that preceded it.

## 8.3 AMS Summer Meeting, 1904

The Columbian Exposition of 1893 exerted a powerful influence on the community of American mathematicians. Another World's Fair was held in 1904, again in today's Midwest, but this time in St. Louis. It too sponsored a set of academic congresses to accompany the usual exhibitions and entertainment that attract substantial crowds. However, by 1904 American mathematicians could take advantage of significant developments that had occurred in the intervening years. Back in 1893 the only organization for American mathematicians was the New York Mathematical Society, then mainly restricted to those living close to New York City. Eleven years later, however, the burgeoning American Mathematical Society could hold its annual summer meeting in conjunction with the St. Louis Congress, headquartered at the Inside Inn on the exposition grounds. We examine the AMS meeting before discussing the Congress.

Altogether, 39 AMS members attended the meeting, not including Henri Poincaré and Gino Fano, who were "present by special invitation" [**246**, p. 55]. In addition, another ten participants who delivered papers but did not belong to the Society were in attendance, as were perhaps five others who attended the St. Louis Congress that commenced two days later. Poincaré was one of twenty-one different speakers who collectively delivered twenty-four lectures at the AMS meeting in two-hour sessions held at 10 a.m. and 2 p.m. on Friday and Saturday, September 16 and 17, 1904. In contrast with modern meetings, no sessions ran concurrently; all were conducted in the Washington University library. Poincaré's talk dealt with closed geodesics on a closed convex surface and, as was his custom, he asserted a conjecture: there must be at least three such geodesics, it being known at the time that at least one must exist. Also characteristic of Poincaré, he provided geometric insight that emboldened such a pronouncement. In this way, he resembled the star of the Chicago Congress, Felix Klein, who relied heavily on geometric intuition in almost all his work.

Two other papers were submitted for the AMS meeting by foreign mathematicians unable to attend, so they were read by E.H. Moore: *Sur les opérations linéaires* by the 25-year-old Frenchman Maurice Fréchet, and "On a method of dealing with intersections of plane curves" by the Englishman Francis Macaulay. Fréchet's paper was particularly relevant for the times because it underscored the very recent work of his countryman Henri Lebesgue. Regarding a critical part of the paper, Fréchet wrote, "the function $K(y)$ is integrable in Riemann's sense or in the extended sense explained by Lebesgue in his recent *Leçons sur l'intégration*" [**246**, p. 61].

Five highly regarded Americans submitted papers for the AMS meeting, three from Chicago and two from Harvard. Leonard Dickson presented two papers on group theory; one was read by title only. The full title of the other describes its contents: "Explicit exhibition of all the subgroups of orders the three highest powers of $p$ in the group $G$ of all $m$-ary linear homogeneous transformations modulo $p$." Oswald Veblen did not attend the meeting so his paper outlining a proof of the Jordan Curve Theorem (that a simple closed curve decomposes a plane in which it lies into two parts) was read by Gilbert Bliss. Veblen was in the process of revising his dissertation, with the aid of fellow students Nels Lennes and R.L. Moore, for publication in the *Transactions of the AMS*, where the paper from this AMS meeting also appeared. Dickson and Veblen were recent additions to E.H. Moore's staff at Chicago. Gilbert Bliss had received his doctorate under Oskar Bolza just four years earlier and had gone to Göttingen to study with Klein and Hilbert for a year in the meantime. He was on the faculty at Missouri for only that one year (1904–1905), when he read his paper giving necessary and sufficient conditions for a certain function from the calculus of variations to be integrable.

The two Harvard mathematicians, both holding doctorates from German universities—Maxime Bôcher (Göttingen) and Edward Huntington (Strasbourg)—read papers on the foundations of mathematics. Bôcher discussed two paradoxes, a famous one due to Bertrand Russell and another due to Jules Tannery. Huntington concentrated his remarks on the independence of postulate systems, such as the postulates for a group proposed by E.H. Moore. Huntington expanded upon his lecture at the St. Louis Congress one week later and subsequently published it in the *Transactions of the AMS* [**278**].

Somewhat surprisingly, mathematicians from the host institution, Washington University, did not participate in the program at the AMS meeting. Nonetheless, those from the University of Missouri were actively engaged in organizing the event and in delivering papers. Earle Hedrick, a student of David Hilbert, had come to Missouri from Yale one year earlier as professor and chair of the department. He became intimately involved in almost all aspects of the tandem AMS and Congress events at once. In addition to presenting two papers at the AMS meeting, one on the calculus of variations and the other on the multitude of derivative notations in different textbooks on differential equations, Hedrick publicized his university's collection of physical models that had been constructed mostly by his colleague Louis Ingold. The exhibit of models was no doubt inspired by Felix Klein's demonstration at the Chicago Congress of 1893 and came on the heels of an extensive exhibition of some 300 models promoted by 25 exhibitors at the 1904 International Congress of Mathematicians at Heidelberg [**538**, pp. 199–200]. The *Bulletin of the AMS* report on the St. Louis meeting provided details of models [**246**, p. 56]:

> At the close of Friday's session an excursion was made to the palace of education, where Professor Hedrick explained the exhibit of the University of Missouri, in particular certain mathematical models constructed by advanced students in his department last year. Of marked interest was a model made by Mr. Ingold to accompany his paper … [It] gave in red and blue wire a large number of lines representing real and imaginary points of a real circle. Other models related to geodesic lines, subgroups of the modular group, and analysis situs.

Another speaker at the AMS meeting was Henry White, who had organized the Evanston Colloquium by Felix Klein eleven years earlier. White was still at Northwestern University but would move to Vassar College the next year. He spoke on quartic and quintic surfaces that admit infinitesimal collineations.

Three of the remaining talks concerned the American specialty of group theory: George Miller (then at Stanford, later a mainstay at Illinois) spoke on a theorem of Burnside on subgroups of abelian groups. John Wesley Young spoke on congruence subgroups of modular groups. Young had just completed his Cornell PhD with a dissertation probably written under the direction of William Benjamin Fite (1869–1932), then at Cornell but later at Columbia. Fite himself delivered the third talk on successive commutator subgroups.

Three other speakers deserve mention. James Byrnie Shaw (1866–1948), of Milliken University, was granted a PhD in 1893 from Purdue University, which did not award another doctorate in mathematics until 1939. A founding member of the Chicago Section of the AMS in 1896, Shaw was ranked among the top ten most active members of the section up to 1923. His paper at the 1904 St. Louis meeting, "Composition of a linear associative algebra," dealt with a topic he covered extensively four years later with the book *Synopsis of Linear Associative Algebra: A Report on its Natural Development and Results Reached up to the Present Time*. A 1929 book of an entirely different nature is also of interest here, *Freshman Algebra*. A reviewer summarized it [**548**, p. 388]:

> Shaw's *Freshman Algebra* is a text to be used primarily by women's divisions in college algebra. The book is not intended to be used in the usual courses in algebra where the subject is applied in as many ways as time permits. It is the author's purpose to present algebra devoid of

> applications, stressing the beautiful and the aesthetic. Poetry, rather than utility, furnished the background for the course. … The book is not a mathematics text in the ordinary sense; it is rather a mathematical companion for a course of study in the purpose, beauty, and mission of mathematics, or, more specifically, algebra.

It is hard today to imagine special mathematics offerings for women, and knowing of Shaw's many activities with the Chicago Section of the AMS, it is equally difficult to imagine the author holding the opinion that women were not able to engage in serious mathematics. A prime counterexample would have been his contemporary Ida May Schottenfels, the second most active woman mathematician (behind Charlotte Angas Scott) in America up to 1906. Schottenfels earned this accolade by publishing three papers and delivering seventeen lectures at AMS meetings during the period 1891–1906, even though her highest degree was a master's (from the University of Chicago in 1896). Her paper at the St. Louis meeting, "On a set of generators for certain substitution and Galois field groups," would seem to suggest that some women were expert in the highest levels of research in algebra.

The remaining speaker at the AMS meeting is yet another fascinating American mathematician. At the time of the AMS meeting, Harry Schultz Vandiver (1882–1973) was a 21-year-old high-school dropout (from the celebrated Central High School in Philadelphia) who had already established a reputation based on a paper he wrote with another youngster, George Birkhoff.[6] Vandiver began attending graduate courses at the University of Pennsylvania in the fall of 1903 while working as a customs-house broker and freight agent for his family's firm in Philadelphia, but he never finished the Penn program either. In fact, the first degree he ever earned was an honorary doctorate that Penn bestowed on him in 1946 (at age 63). His AMS paper, "On reduction algorithms for the solution of the linear equation in a finite field," provided a new algorithm not only for solving linear equations in a finite field but also for fields whose residue classes (of a prime ideal in an algebraic field) are considered in connection with a linear congruence.

Vandiver left the family business to serve as a yeoman with the US Naval Reserve (1917–1919). Upon discharge from active duty, he was persuaded by George Birkhoff to enter academia, so he accepted an instructorship at Cornell, where he spent five fruitful years, three supported by research grants. In summers, he worked in Chicago editing Chapter 26 in Volume II of Leonard Dickson's voluminous *History of the Theory of Numbers*. By 1924 Vandiver had published over twenty papers, which earned him an offer from Milton Porter as associate professor at the University of Texas in Austin. Vandiver remained at Texas until retirement in 1966, having been made a full professor of pure mathematics in 1925. However, in 1947 he switched to the department of applied mathematics and astronomy as distinguished professor because of a nasty fight with R.L. Moore—he continued to practice the most abstract form of number theory.

Throughout his career, Vandiver received numerous grants and senior fellowships that allowed him to live in various places around the US when most mathematicians were content to stay put. He won the coveted Cole Prize in the theory of numbers in 1931 for his proofs of several special cases of Fermat's Last Theorem. In 1955 he

[6]Most of the material on Vandiver is based on a faculty Memorial Resolution written by four Texas colleagues at the time of his death.

coauthored a paper proving, with the aid of a computer, that if Fermat's Theorem was false, then the exponent would have to be at least 4002.

How was this peripatetic mathematician able to travel so frequently? The Vandivers (including wife Maude Folmsbee and son Frank) never owned a house. Instead, they rented apartments and lived in the homes of other faculty members who were on leave. Vandiver also maintained a permanent room at the Alamo Hotel in Austin, where he retreated for research while listening to his huge collection of classical recordings. However, by the mid-1950s his manner of working far into the night without eating brought him to the brink of physical collapse, whereupon he stopped accepting most outside offers to give lectures, write reviews, or referee papers for journals. His last public appearance was in 1961 when, at age 78, he was invited to deliver the principal address at the Texas Section of the MAA; over 200 people attended the meeting. The editors of the *Journal of Mathematical Analysis and Applications* dedicated the 1966 volume to him, and many of his former students, colleagues, and friends responded with papers in his honor. That year, at age 84, he resigned his modified service appointment to become emeritus.

Among the other speakers at the AMS meeting, only one gave two talks. David Raymond Curtiss (then at Yale but moved to Northwestern for the rest of his career) read the papers *Sur certains théorèmes de la valeur moyenne* and *Sur la théorie des fonctions hypergéométriques.* Four who each presented one paper were Lewis Darwin Ames (Missouri), "Supplementary communication on the division of space by a closed surface"; Thomas John Bromwich (from Ireland), "The classification of quadrics"; Edward Brind Escott (Michigan), "The expression of a quadratic surd as a continued fraction"; and Louis Ingold (Missouri), "Real representation of the real and imaginary portions of a plane locus." Details of these works can be found in the *Bulletin of the AMS* report by Haskell and White on the entire AMS meeting [**246**].

## 8.4 Meet Me in St. Louis, Louis

The International Exposition of 1904 in St. Louis sponsored a set of academic congresses to accompany the usual exhibitions and fairs that draw substantial crowds. (Just as the 1893 Columbian Exposition in Chicago had). The St. Louis congresses met on the grounds of Washington University, but they did not have the same effect on that institution as the earlier one did on Chicago, even though the Columbian Exposition did not make use of buildings on the new campus. However, the 1904 gathering is of historical interest both because of the quality of the speakers on the program, with its inherent ranking of the most accomplished mathematicians in the land, and for the way in which mathematicians in the new century could take advantage of the AMS.

St. Louis was teeming with activity in 1904. Named for Louis IX, the city was established in 1764 on a land grant from the King of France as a fur-trading post because it was located near the confluence of the Mississippi and Missouri Rivers. The town gained fame as the starting point for the Louisiana Purchase Expedition of Meriwether Lewis and William Clark in 1804. The population of St. Louis grew steadily; by 1900, it was the nation's fourth largest city and a major manufacturing center. To commemorate the centenary of the Louisiana Purchase, the city hosted a World's Fair, known officially as the Louisiana Purchase Exposition.