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Introduction

You are off on a road trip to New York City and hope to get there before rush hour. Driving at a law-abiding 65 mph, you notice that every driver around you is traveling 75 mph. You wonder how much sooner you would arrive if you joined the flow.

You usually bake one big round loaf of bread on a sheet pan. Tonight, you are having guests and you decide to split that loaf into eight equal sized rolls, each half as big as the original in every dimension. Will your rolls fit on the same sheet pan?

Remodeling your somewhat oddly shaped bathroom, you are trying to calculate the number of square feet of ceramic tile to order for the floor.

Out on a hike, you wonder if it is faster to hike a loop-trail clockwise or counterclockwise, or is it all the same?

What are my odds of contracting the flu this season if I get vaccinated?

How did Penn and Teller do that card trick on TV?

The contestant on *Jeopardy!* just made a strange wager in *Final Jeopardy!* Was it correct?

What is a good score in a March Madness office pool bracket for the NCAA basketball tournament?

Is it a smart deal to pay for extra collision insurance on my new car?

How many cubic feet of potting soil do I need to buy to fill my planter?

Does it save me money to shop at Costco and buy in bulk? Are there downsides?

How certain can I be of the weather forecast, the presidential polls, or other scientific predictions?

Is there a mathematical way to figure out the lost combination for a lock?

Mathematical ideas are all around us. Mathematics can be found when we hike in the woods, play games, watch TV, eat at restaurants, do our taxes, visit the library, go to a party, read the newspaper, converse at dinner, surf the Internet, or drive along the highway. I don't mean the stuff you do at school: arithmetic practice, factoring expressions, solving linear and quadratic equations, or writing geometric proofs. Rather, I refer to the skeptical inquiry that asks how and why things work, the process of experimenting, exploring, conjecturing, and discovering. Estimating, analyzing, calculating, understanding, and uncovering truths are all mathematical pursuits.

As a mathematics teacher and professor, I spend my life trying to make math relevant and interesting to people of all ages who have no particular passion for mathematics. Students and friends often challenge me with questions like:

"Why do I need to know this? Where does this come up in *real life*?"

The short answer is "everywhere." Mathematics is all around us in the mundane and the profane, at work and at home. Yes, there are deep theorems with applications to all sorts of engineering, science, public policy, climate change, polling, and the like, but the beauty and joy of mathematics is seeing it right in front of one's eyes. And, when you start to notice mathematics in everyday life is when you have a chance to appreciate and even enjoy it.

Many people look for mathematics in all the wrong places. You do not need to read the reports of an engineer, the papers of a scientist, school textbooks, or watch YouTube lectures to find mathematics. You need only to look around you with the right perspective. You may find my perspective nerdy or downright peculiar, but I welcome you to share it and I hope you will find it more charming than disarming.

The mathematical level of the material varies; one person's "fun and easy" may be another's "impenetrable and incomprehensible." So, if you feel you want to bail out when the going gets rough, then put the book down for another day or just skip to a different section. The chapters are independent, so it's perfectly okay to jump around.

The topics in the book also vary; the only sure commonality is that they suit my taste. If you get bored with one piece, move to another. I am confident that you will find something to your liking.

This book reveals mathematics in every corner of life: in conversation, in the newspaper, in nature, in books, in history, in plain sight, whether simple, silly, or complex. I hope to convey a child-like excitement and appreciation of the mathematics that surrounds us. Where is mathematics in real life? *Here, there, and everywhere*. Let's see.

13

Happy Birthday

13.1 Reverse Ages

Recently, we celebrated my son's 26th birthday and we noticed he was the reverse of my 62 years. We wondered how common this was and whether it had ever occurred before. Did it, or would it, happen with my other children? What are necessary and sufficient conditions for two people to have their ages be the reverse of each other? If it happens once, will it happen again?

My son's friend Avi quickly answered the last of these questions. He noticed that if two people have reverse ages, then eleven years later, it will happen again. Adding one to each digit maintains the reversal. AB and BA become [A+1][B+1] and [B+1][A+1]; for example, 35 and 53 become 46 and 64. This breaks down once you get over 100, but we felt safe in ignoring that case. The entire group followed this reasoning easily. We were getting somewhere.

What about necessary and sufficient conditions? My youngest son noticed that the smallest case of a reversal is the pair of ages 01 and 10, and as Avi pointed out earlier, the ages of a one-year-old and a ten-year-old would reverse again every eleven years at 12 and 21, 23 and 32, 34 and 43, and so on. Similarly, my oldest son pointed out that the next smallest pair of reversed ages is 02 and 20, which repeats again at 13 and 31, 24 and 42, 35 and 53, and so on. My wife, who tends to take everything in before she speaks, pointed out that the next case was 03 and 30, and moreover, she noticed that the differences between these three pairs of ages: 01 and 10, 02 and 20, and 03 and 30, are 9, 18, and 27 respectively.

I was in seventh heaven. There is nothing I enjoy more than friends and family actively engaging in problem solving and mathematical discovery no matter what the level of complexity. After some thought, I realized she had figured it all out. The difference between the ages X0 and 0X is exactly $10X - X = 9X$. Furthermore, given any two ages that are reverses of one another, we can repeatedly subtract 11 from the younger year until it is in the single digits. Thus, the earliest occurrence of the reversal happens when the younger person is 0X years old, and the older is X0, like 03 and 30, or 06 and 60. Therefore, two people will be reverse ages of each other every eleven years if and only if their ages differ by a multiple of nine. Indeed, I am 36 years older than my middle son; and since I am 33 and 39 years older than my other two sons, I will never be the reverse of their ages.

13.2 Jigsaw Puzzles

One of the gifts my son received for his birthday was a jigsaw puzzle. It's interesting to watch people solve jigsaw puzzles. They use all sorts of ad-hoc algorithms. My wife's parents are practiced experts at jigsaw puzzles and patiently solved 18 of them when they were quarantined during the COVID-19 pandemic. Here is what I learned by watching them.

There is often some preprocessing. This typically consists of sorting the straight edge pieces from the rest and then sorting again by similar colors. After this preprocessing, there are two contrasting algorithms that often occur simultaneously.

> Algorithm 1: Pick up a piece that looks easy to place. Try to guess where the piece belongs in the larger puzzle by examining the picture of the puzzle that appears on the box.
>
> Algorithm 2: Look for pieces that connect one to another due to similar color or fitting shape and build these pieces into clumps. Small clumps combine into big clumps, and these eventually connect into the larger puzzle.

Interestingly, as a computer scientist, I noticed that these two algorithms are respectively reminiscent of Prim's and Kruskal's algorithms for solving a famous graph theory problem called *minimum spanning tree*, but that is not the point here. Minimum spanning tree algorithms are not everyday life for most people. Real life is the family working together on a puzzle.

My son opened his gift and the family worked together on the puzzle for about two hours. When they had placed about half the pieces, I wondered how many more hours it would take to solve the puzzle. Or, in general, what is the relationship between the number of pieces in a puzzle and time it takes to solve it?

Of course, different kinds of puzzles will take longer than others. A single uniform color puzzle, for example, will be harder than a puzzle with complex patterns, and the shape of the pieces plays a role as well. However, I was interested only in the relationship between solution time and puzzle size. That is, given a *specific* puzzle, how does the number of pieces that remain unplaced relate to the time needed to finish solving the puzzle?

Surely, two separate puzzles would take twice as long as one, and three would take three times as long, and so on. I had a feeling, however, that adding twice as many pieces to a single puzzle would necessitate much more than twice the time. You might think of it as magnifying the picture and using twice as many pieces to cut it up. My family agreed that it is safe to assume that doubling the pieces of a puzzle will more than double the time needed to solve it. The question is how much more?

The world record for a 250-piece puzzle is 13 minutes and seven seconds, completed by Deepika Ravichandran on June 9, 2014. Joellen Beifuss set the record for a 500-piece puzzle in 1984, completing it in 54 minutes 10 seconds. In contrast, typical average times for 250, 500, and 1000-piece puzzles are: 1 hour, 4 hours, and 9 hours, respectively.[1] I could not find any single-person records for 1000 pieces, but there is a record for a team of two. This data is summarized in the figure.

Number of Pieces	Average Solution (Hours)	World Record (minutes)
250	1	13:07
500	2-7	54:10
1000	10-30	61:29 (pair)

The data suggest that when you double the number of pieces in a puzzle it takes roughly four times as long to solve it. The numbers are not perfect, but it seems like a reasonable conjecture. Let $T(x)$ be the time needed to solve a puzzle with x pieces. Restating our conjecture, we have $T(x) = 4T(\frac{x}{2})$.

Replacing x with $\frac{x}{2}$ implies that $T(\frac{x}{2}) = 4T(\frac{x}{2^2})$. Substituting $4T(\frac{x}{2^2})$ for $T(\frac{x}{2})$ in $T(x) = 4T(\frac{x}{2})$, results in

$$T(x) = 4^2 T\left(\frac{x}{2^2}\right)$$

Repeating this process again, we see that $T(\frac{x}{2^2}) = 4T(\frac{x}{2^3})$, and therefore $T(x) = 4^3 T\left(\frac{x}{2^3}\right)$. Continuing in this way r times, we get

$$T(x) = 4^r T\left(\frac{x}{2^r}\right)$$

Sooner or later, we will reach an r where $x = 2^r$, and then $T(x) = 4^r T(1)$. $T(1)$ is some constant time c, independent of x, so we have $T(x) = c4^r$ for some

[1] `https://journeyofsomething.com/blogs/news/how-long-does-it-take-to-finish-jigsaw-puzzles-of-different-sizes`, `https://www.seniorcare2share.com/how-long-does-it-take-to-do-a-1000-piece-puzzle/`

constant c. Finally, if $x = 2^r$ then $4^r = 2^r \times 2^r = x^2$. So, $T(x) = cx^2$. Thus, the time needed to solve an x-piece puzzle is quadratic in x.

This makes some intuitive sense because in the worst-case scenario, using Algorithm 1, one needs to compare each puzzle piece to every other piece until all the pieces are placed. The first of the x pieces is compared to $x - 1$ pieces, and after this piece is placed in the puzzle, the next piece is compared with the remaining $x-2$ pieces, and so on. This amounts to $(x-1)+(x-2)+\ldots+3+2+1$ total comparisons. By pairing up the numbers at the ends of this expression one can see the sum consists of $\frac{x-1}{2}$ pairs each of which sums to x. And that is $\frac{x(x-1)}{2}$ comparisons of pieces, which is quadratic.

If we are correct that $T(x) = cx^2$, then $T(2x) = c(2x)^2 = 4cx^2$. This confirms that doubling the number of pieces will make the project four times as long. And, tripling the pieces, $T(3x) = c(3x)^2 = 9cx^2$, multiplies solution time by nine. You can test this conjecture by measuring your own solution times for jigsaw puzzles with different numbers of pieces.

13.3 Social Distancing

Trying to celebrate a birthday during the COVID-19 pandemic of 2020 was difficult. The participants had to wear masks and stay socially distant, meaning about seven feet away from each other. Six of us were celebrating in a nine-foot by twelve-foot room, but it seemed impossible for all of us to keep seven feet apart. One exasperated child exclaimed "this is impossible!" She was right. Prove that if you place six people into a nine-foot by twelve-foot rectangular room, there must be at least one pair of people who are closer than seven feet apart.

The first proof is due to my nephew David, a budding mathematician and computer scientist.

His idea is that the best you could hope to do is to have four people stand as far as possible from each other in the four corners of the room. In Figure 13.1, circles of diameter seven are drawn around each person, and a two by five white rectangle is drawn in the middle of the room, so that if you extended the white rectangle's four edges, they would all be tangent to the circles. David claims that you cannot place any person in the shaded region outside the white rectangle because the distances are all too close to the corners. Indeed, the furthest you can get away from all four corners is halfway along the top or bottom of the white rectangle. Pythagoras's theorem tells us that this point is $\sqrt{6^2 + 3.5^2} \approx 6.95$, away from the nearest corner, a tad too close.

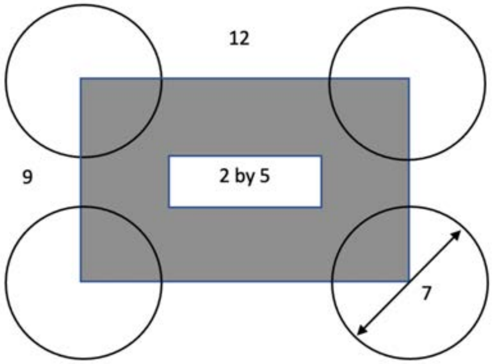

Figure 13.1. David's Proof: Six People Seven Feet Apart is Impossible in a 9 by 12 Room

David concludes, therefore, that the last two people, the fifth and sixth, need to be placed inside the white rectangle. However, there is not enough room in the white rectangle to keep those two people seven feet apart. The longest distance between any two points in the white rectangle is the diagonal, which has length equal to $\sqrt{2^2 + 5^2} \approx 5.38$, which is way too close.

It is a very nice proof, but the part about "the best you could hope to do is to have four people stand as far as possible from each other in the four corners of the room" is a little fuzzy. Maybe we could do better by moving those four people off the corners? We cannot do better, and I believe that his proof can be made rigorous, but the best part of his proof is that it feels intuitive and accessible. We'll try to "rigorize" his proof later.

Meanwhile, here is a different proof that is remarkably clever, but leaves you with the feeling of "who would ever think of that?" The proof is completely rigorous but not as accessible as David's "four people in the corners" proof. This alternative proof uses an idea called the pigeonhole principle, which states the obvious fact that six pigeons placed into five pigeonholes necessitates at least one hole with two cozy pigeons.

Consider Figure 13.2 showing the nine-by-twelve-foot room divided into five sections, where the numbers shown are in units of feet.

Since we have six people placed into five sections, by the pigeonhole principle, there must be one section with at least two people. Now notice that the longest distance between any two points in each section can be no more than the dashed line in the figure. The length of this line is $\sqrt{6^2 + 3^2} = \sqrt{45} \approx 6.71$, which is strictly less than seven feet. That's it!

A very clever proof indeed. There are no assumptions here about the corners of the room, just a very simple, complete, and elegant argument. I am not sure

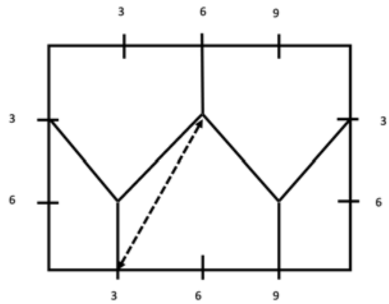

Figure 13.2. A Clever Proof

how the composer came up with the shapes, but I think this proof might be easier to find if I originally asked to prove that some pair of people had to be within $\sqrt{45}$ feet of each other, rather than seven.

After examining this clever pigeonhole proof, it may dawn on you that David's proof is somewhat similar. David's proof can be made completely rigorous by partitioning the room into four quarter-circles each of radius seven (twice the diameter of the ones in Figure 13.1) centered on the corners, and a two by five rectangle in the middle. David's partitions overlap, but the idea is otherwise the same. With six people, one or more of the circles or the rectangle must contain at least two people, and there is not enough room for two people to stay seven feet apart in any circle of diameter seven, or in a rectangle of two by five.

13.4 Socially Distant Seating

Here is another scenario motivated by social distancing. In the fall of 2020 during the COVID-19 pandemic, my college had a rule that no person may sit directly next to anyone else. That is, there must be at least one empty seat between any two people. Students arrive and take their seats at random times, so I set up a "rule" that when someone enters, they must choose a seat as far away from anyone else as possible. Imagine 25 permanent chairs in a single row, a simplified version of my classroom. If chairs 1, 7, and 20 are occupied, a new arrival would need to sit in chair 13. This keeps at least five empty seats between the newcomer and anyone else already seated. Chair 14 would be just as good, but 13 is smaller, and we might as well break a tie in favor of the lower numbered chair.

With this rule in place, the only real choice is where to seat the very first student. For example, if the first student sits in the first chair, then the second student is placed in chair 25, and the order of seating proceeds like this: 1, 25, 13, 7, 19, 4, 10, 16, 22. Now, no more students can be seated without having someone next to another person.

If the first student chooses the middle seat, how many students can eventually be seated? The sequence in this case is: 13, 1, 25, 7, 19, 4, 10, 16, 22, and once again no more students fit. Indeed, this sequence of seats is the same as before except for the first three seats. Can you do better than nine students with a better choice of starting seat? After all, if I dispensed with my rule, and simply closed off the even-numbered seats, I could fill all the odd-numbered seats, and accommodate thirteen students.

It turns out that starting with seat number seven works out well. The sequence is: 7, 25, 16, 1, 11, 20, 4, 9, 13, 18, 22. Starting with seat seven seats eleven people. Can we do any better? We can. Starting with seat nine, gives: 9, 25, 1, 17, 5, 13, 21, 3, 7, 11, 15, 19, 23. This is as good as it gets, filling all thirteen odd numbered seats. Starting at 17 would also fill the same seats, but in a different order: 17, 1, 9, 25, 5, 13, 21, 3, 7, 11, 15, 19, 23. Indeed, after the first four students are seated, the seating order is the same as when you start with nine.

It appears that the best thing to do is to leave a power of two empty seats on either side of the starting seat. For 25 seats, this means leaving eight or sixteen empty seats around the starting seat. That means starting with seat nine, leaving eight open seats to the left and sixteen to the right; or you can start with seat seventeen, leaving sixteen open seats to the left and eight to the right. Does this "power of two" strategy work with any number of seats in a line? Can you prove it does?

You might be interested in exploring what happens if we add a second row to this scenario. Assume that the distance between the two rows is the width of one seat, and add an extra rule that one person may not sit directly in front or in back of another. As before, each new student must sit as far as possible from everyone already seated. Where should we seat the first person in order to maximize the number of people seated?

13.5 Craps at the Casino

For my oldest son's 21st birthday, my brother-in-law and I took him to a local casino and tried to teach him the wisdom of not gambling. We taught him the game of craps and explained all the different ways to bet, including the ones that minimized his expected losses. In other words, we taught him how to play for as long as possible before losing all his money.

Each of us gave him $50, for a total of $100, and we agreed to stand by the table to help him remember how to bet. If he somehow got lucky and managed to double his money, then we would pick up and leave and he makes $200 for his birthday. Otherwise, we stay until he loses all his money, and we take him out to dinner. He got lucky and eventually accumulated $200, doubling his stake. His success felt effortless to him, if not inevitable, so he glanced over to us at the end of the table, asked if he could play just one more time. We smiled, shook our heads no, and walked out. He followed us with his $200, and we still paid for his dinner.

If my son had continued to play, then he would have eventually lost all his money, the $100 he earned plus his original stake. Like any vice, gambling can be an occasional pleasant pastime only if you set limits and stick to them.

As a source of income, gambling is a bust, but it can be cost effective as a form of entertainment. Assume you just wanted to play for as long as possible, until you lose all your money. How much time would it take on average to lose $100 at craps, betting $10 each game?

Let's estimate. There are two considerations that affect the length of our stay in the casino.

(1) The number of games played on average until $100 is gone.

(2) The average time to play each game.

First, let's calculate the average expected number of games. To do this, we need to review the rules of craps. Craps is a dice game. If you roll a 7 or 11 you win. That's called a *natural*. If you roll 2, 3, or 12, you lose. That is called *craps*, the name of the game. Any other roll, 4, 5, 6, 8, 9, or 10 is called your point and you keep rolling until you roll a 7 or you roll your point again. If the point comes again before a 7 then you win, otherwise you lose. When you roll a point, no other subsequent values matter except the point and 7.

What are the chances of winning a game of craps? We can add up all the possibilities. The chances of rolling a 7 are $\frac{6}{36}$, and the chances of rolling an 11 are $\frac{2}{36}$. So, the chances of winning with a natural roll are $\frac{8}{36} = \frac{2}{9}$. The chances of winning when rolling a 4 are $\frac{3}{36}$ for the first 4, times the chances of rolling a 4 before a 7. The chances of rolling a 4 before a 7 are the same as the chances of rolling a 4 on the condition that you have rolled a 4 or 7. There are nine ways to roll 4 or 7 and three ways to roll a four, so the chances of rolling a 4 before a 7 are $\frac{3}{9}$. Thus, the chances of winning a game of craps starting with a 4 are $\frac{3}{36} \times \frac{3}{9} = \frac{1}{36}$. The exact same calculation can be done for 10. For 5 and 9, the chances of winning are each $\frac{4}{36} \times \frac{4}{10} = \frac{2}{45}$. And, for 6 and 8, the chances of winning are each $\frac{5}{36} \times \frac{5}{11} = \frac{25}{396}$.

Adding it all up results in:

$$\frac{2}{9} + \frac{1}{36} + \frac{1}{36} + \frac{2}{45} + \frac{2}{45} + \frac{25}{396} + \frac{25}{396} = \frac{244}{495} \approx 0.493.$$

This means that, on average, playing 1000 games results in 493 wins and 507 losses. Betting $10 each game, using the so-called *Pass Line* bet that pays even odds, means an average loss of $140 over 1000 games, or 14 cents per game. At this rate, you will expect to lose $100 after about 714 games.

We now have an estimate of how many games the \$100 is expected to last on average. It is 714. Incidentally, serious craps players know that there are better betting options with different odds that let you lose slightly less that 14 cents per game and thereby last a little longer than 714 games, but 714 is a reasonable estimate for casual players.

To finish our estimate of the time it will take to blow \$100 at the craps table betting \$10 a game, we need to calculate how much time a single game of craps will last. I did a random sampling at a busy craps table. A single throw takes about forty seconds, including the throw and the time for the croupier to collect the dice, pay off bets, take new bets, and get you ready to throw again. We need to calculate the expected number of throws in a single game.

Let's add up all the cases in a similar manner to how we calculated the expected chances to win. For 2, 3, 7, 11, or 12, the game lasts exactly one throw. This happens 12 out of 36 times. For 4, the game lasts 1 + the number of expected rolls to get a 4 or 7. Rolling a 4 or a 7 happens 9 out of 36 times, so we expect that the average number of rolls needed to see a 4 or 7 is $\frac{36}{9} = 4$. This is the same for 10. For 5 and 9, the expected number of rolls is $1 + \frac{36}{10}$ for each. And, for each of 6 and 8, the expected number of rolls is $1 + \frac{36}{11}$.

Adding it all up, we get:

First Roll	Chances	Average Number of Rolls	Chances × Number of Rolls
2, 3, 7, 11, 12	$\frac{12}{36}$	1	$\frac{1}{3}$
4, 10	$\frac{6}{36}$	$1 + \frac{36}{9}$	$\frac{5}{6}$
5, 9	$\frac{8}{36}$	$1 + \frac{36}{10}$	$\frac{2}{9} + \frac{4}{5}$
6, 8	$\frac{10}{36}$	$1 + \frac{36}{11}$	$\frac{5}{18} + \frac{10}{11}$

Summing up the last column gives the average expected number of dice throws in a game of craps as:

$$\frac{1}{3} + \frac{5}{6} + \frac{2}{9} + \frac{4}{5} + \frac{5}{18} + \frac{10}{11} = 3 + \frac{62}{165} \approx 3.376$$

Estimating forty seconds a throw, times 3.376 throws per game, times 714 games, gives approximately 26.8 hours. On average, you can expect \$100 to afford you lots of entertainment before you go bust playing craps. And that's only playing time; it doesn't include time in between games to chat or to order food and drinks. This works out to about \$4 an hour, a favorable rate compared to other forms of entertainment such as the theatre, a movie, or live professional sports. We were in the casino only about three hours before my son doubled up and we walked out.

Abiding by our stipulation that we walk out when we either double up or go broke, and that we bet the same amount for every game of craps, we can use a sophisticated tool called an *absorbing Markov chain* to answer the following theoretical questions and compare the answers to what happened to us at the casino.

(1) What are the chances we will go broke versus the chance of doubling up?

(2) What is the expected number of games played before we either double up or go broke? (This should be significantly less than 714, the expected number of games until we eventually bust.)

The details of absorbing Markov chains are beyond the scope of this book. Not every mathematical idea is casually accessible. Suffice it to say that the answers to my questions have to do with the inverse of the following matrix.

1	-.493	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-.507	1	-.493	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-.507	1	-.493	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-.507	1	-.493	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-.507	1	-.493	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-.507	1	-.493	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-.507	1	-.493	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-.507	1	-.493	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-.507	1	-.493	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-.507	1	-.493	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-.507	1	-.493	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-.507	1	-.493	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-.507	1	-.493	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-.507	1	-.493	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-.507	1	-.493	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-.507	1	-.493	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-.507	1	-.493	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-.507	1	-.493
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-.507	1

I did the work and here are the answers. We go broke about 57% of the time and double up about 43%. My son was lucky. His chances of doubling up were only 43%.

The expected number of games played before going broke or doubling up is a little over 99 games. At forty seconds a throw, 3.376 throws per game, the time to play 99 games is about three hours and forty minutes. My son took three hours, somewhat less than expected. Lucky 21.

13.6 Social Media Musings

Adam, a youngish friend of mine, recently turned 40. His adoring wife Michelle posted on Facebook how happy she was to have spent sixteen years together with him, exactly 40% of his 40 years. She proceeded to sing his praises in a devoted way that convinced me they would spend many more happy years together. I wondered whether there would be another time in Adam and Michelle's future when she could say she spent $x\%$ of his x years together with him.

Letting x be Adam's age. Then $x\%$ of x is $\frac{x}{100} \times x = \frac{x^2}{100}$ years. Since he turned 40, they have been together for 16 years. Now if this is to happen again, say in r years, they will have been together $\frac{x^2}{100} + r$ years, and that must equal $x + r$ percent of his age $x + r$. Or,

$$\frac{x^2}{100} + r = \frac{(x+r)^2}{100}$$

This reduces to a real-life quadratic equation, $r^2 = 100r - 2xr$, which, conveniently, can be solved without resorting to the quadratic formula. Assuming that r is nonzero, we divide through by r and get $r = 100 - 2x$.

Since Adam's age is 40, x is 40, and thus r is 20. So, in 20 years, once again the two will have been together for $x\%$ of his x years, when he is 60 years old. At that time, Adam and Michelle will have been together for 60% of 60, or 36 years. Happy anniversary! After 60, she can never use this cute time marker again; she will need to think up some other clever thing to say. She is very witty so I have no worries she will come up with something even better. Could she have used this same remark before he was 40? Nope. Maybe you can prove why.