Problem Set 1

Opener

This video shows a deck of cards being shuffled eight times:

`http://go.edc.org/8shuffles`

Figure out what you can about this.

Important Stuff

1. Does the perfect shuffle work for other deck sizes? If not, why not? If so, what stays the same and what changes?

2. Evelyn is thinking of a positive integer, and because she's a math teacher she calls it x. What information would you know about the last digit of x based on each statement?

a. $3x$ has last digit 4
b. $7x$ has last digit 4
c. $4x$ has last digit 4
d. $5x$ has last digit 4

3. **a.** What number is $9 \cdot 10^1 + 9 \cdot 10^0 + 4 \cdot 10^{-1} + 4 \cdot 10^{-2}$?
b. Ben's favorite number is 802.11_{10}. Write it as a sum of powers of 10.

The little 10 here means the number is in base 10. Bases will generally be given when they seem needed, and we'll try not to be confusing.

4. **a.** What number is $1 \cdot 3^3 + 0 \cdot 3^2 + 2 \cdot 3^1 + 0 \cdot 3^0 + 1 \cdot 3^{-1}$?
b. Carol's favorite base-3 number is 2110.2_3. Write it as a sum of powers of 3.
c. Convert 2110.2_3 to base 10.

Surely you have a favorite number in each base, right?

5. Write each number as a decimal. The decimals may terminate or repeat.

a. $\frac{1}{2}$
b. $\frac{1}{50}$
c. $\frac{1}{9}$
d. $\frac{2}{9}$
e. $\frac{9}{9}$
f. $\frac{1}{13}$

6. Carly says, "Here's some numbers, now make them base three."

a. 9 **d.** $\frac{1}{9}$

b. 13 **e.** $\frac{1}{13}$

c. 242 **f.** $\frac{1}{242}$

7. Write each number as a base-3 "decimal". The decimals may terminate or repeat.

For example, $\frac{1}{3}$ is 0.1_3. Try not to call the digits "tenths" or "hundredths"!

a. $\frac{1}{9}$ **d.** $\frac{3}{2}$

b. $\frac{1}{2}$ **e.** $\frac{1}{13}$

c. $\frac{2}{2}$ **f.** $\frac{1}{242}$

Neat Stuff

8. Under what circumstances will a base-10 decimal repeat?

9. Under what circumstances will a base-3 decimal repeat?

10. The repeating decimal $.\overline{002}$ means .002002002 . . . But what number is it? That depends on the *base*! Ace this problem by finding the base-10 fraction equal to $.\overline{002}$ in each given base.

Look up: You did some work on base 3 already.

a. base 3 **d.** base 7

b. base 4 **e.** base n

c. base 5 **f.** base 2?!

11. **a.** Find all positive integers n so that the base-10 decimal expansion of $\frac{1}{n}$ repeats in 3 digits or less.

b. Find all positive integers n so that the base-3 "decimal" expansion $\frac{1}{n}$ repeats in 4 digits or less.

12. Write 13 and 242 in base $\sqrt{3}$ instead of base 3. Hm, what's going on here?

Tough Stuff

13. Aziz has a cube, and he wants to color its faces with two different colors. How many different colorings are possible? By "different" we mean that you can't make one look like the other through a re-orientation.

14. What about edges?

15. **a.** Convert 13 to base $\frac{3}{2}$.
b. Convert 13 to base π.

Table for Problem Set 2

Complete this table with the number of perfect shuffles needed to restore a deck to its original state.

n	# shuffles	n	# shuffles
3	2	19	
4		20	
5		21	
6		22	
7		23	
8		24	
9	6	25	
10		26	
11		27	
12		28	
13		29	
14		30	
15		31	
16		32	
17		33	
18		34	

Problem Set 2

Opener

Can perfect shuffles restore a deck with 9 cards to its original state? If so, how many perfect shuffles does it take? If not, why not?

Split the cards 5-and-4, and keep the top card on top.

Important Stuff

1. Fill in a whole lot of the table on page 6, listing the number of shuffles needed to restore a deck to its original state.

   If you're working in a group, use one handout to record everyone's information.

2. Find the units digit of each annoying calculation, without a calculator.

   The *units digit* of 90210 is 0.

   a. $2314 \cdot 426 + 573 \cdot 234$
   b. $(46 + 1)(46 + 2)(46 + 3)(46 + 4)(46 + 5)$
   c. $71^4 \cdot 73^4 \cdot 77^4 \cdot 79^4$

3. Find all possible values for the units digit of each person's positive integer.

   a. Amy: "When you add 5 to my number, it ends in a 2."
   b. Brandon: "When you multiply my number by 3, it ends in a 7."
   c. Carmen: "When you multiply my number by 6, it ends in a 4."
   d. David: "When you multiply my number by 5, it ends in a 3. Yup."

4. Unlike "base 10", in *mod 10* the only numbers are the remainders when you divide by 10. In mod 10, 6 + 5 = 1 because 1 is the remainder when 6 + 5 is divided by 10.

   This is sometimes called *modular arithmetic*. Clock arithmetic is mod 12.
   That last one says box to the fourth power, by the way.

   Answer all these questions in mod 10.

   a. $2 + 2 = \square$ **d.** $4 \cdot \square = 2$
   b. $3 \cdot 4 = \square$ **e.** $5 \cdot \square = 3$
   c. $\square + 5 = 2$ **f.** $\square^4 = 1$

5. Repeat the previous problem, except this time do the arithmetic in *mod 7* instead of mod 10.

   Good news: there are only 7 numbers in mod 7. Bad news: in mod 7, every Monday is the same.

6. Go back to the big table you worked on in Problem 1. What patterns do you notice?

Neat Stuff

7. Write each fraction as a base-10 decimal.

a. $\frac{1}{5}$

b. $\frac{1}{25}$

c. $\frac{1}{7}$

d. $\frac{3}{7}$

e. $\frac{2}{7}$

f. $\frac{6}{7}$

g. $\frac{1}{13}$

h. $\frac{2}{13}$

8. Write each base-10 fraction as a base-3 decimal. Some of the answers are already given and might be useful in making more!

a. $\frac{1}{13} = 0.\overline{002}_3$

b. $\frac{2}{13}$

c. $\frac{3}{13}$

d. $\frac{9}{13}$

e. $\frac{10}{13}$

f. $\frac{1}{7} = 0.\overline{010212}_3$

g. $\frac{3}{7}$

h. $\frac{9}{7}$

i. $\frac{6}{7}$

9. Write the base-10 decimal expansion of

$$\frac{1}{142857}$$

10. Marvin wonders what kinds of behavior can happen with the base-10 decimal expansion of $\frac{1}{n}$. Be as specific as possible!

11. If $\frac{1}{n}$ terminates in base 10, explain how you could determine the length of the decimal based on n, without doing any long division.

12. What kinds of behavior can happen with the base-3 decimal expansion of $\frac{1}{n}$?

13. Sara and Joe argued about whether or not the number .99999... was equal to 1. Is it? Be convincing.

14. **a.** Suppose $ab = 0$ in mod 10. What does this tell you about a and b?

b. Suppose $cd = 0$ in mod 7. What does this tell you about c and d?

15. **a.** Investigate the base-10 decimal expansions of $\frac{n}{41}$ for different choices of n. What happens?
b. Investigate the *base-3* expansions of $\frac{n}{41}$ for different choices of n. What happens?

The fraction $\frac{n}{41}$ is still in base 10 here, so don't convert 41 to some other number.

16. **a.** Find all positive integers n so that the base-10 decimal expansion of $\frac{1}{n}$ repeats in exactly 4 digits.
b. Find all positive integers n so that the base-3 "decimal" expansion of $\frac{1}{n}$ repeats in exactly 5 digits.

17. Write 223 and 15.125 in base 2. Then write them in base $\sqrt{2}$.

Tough Stuff

18. Investigate shuffling decks of cards into three piles instead of two. What are the options? Does it "work" like it does with two piles?

19. Barbara has an octahedron, and she wants to color its vertices with two different colors. How many different colorings are possible? By "different" we mean that you can't make one look like the other through a re-orientation.

20. What about edges?

21. Find all solutions to $x^2 - 6x + 8 = 0$ in mod 105 without use of any technology. There's probably more.

CHAPTER

2 Facilitator Notes

The facilitator notes are designed to be used as needed. Each session has two components:

1. **Goals of the Problem Set:** here we lay out what the principal ideas of each session are.
2. **Notes on Selected Problems:** we identify a few problems that are worth going over in a whole group discussion.

We will put our emphasis on the main goals of each lesson, drawn from the problems in the "Important Stuff."

Problem Set 1

Goals of the Problem Set

This course starts with a puzzling problem, exploring that problem while developing the necessary mathematics to solve the problem and following some of its extensions. The course will explore modular arithmetic, base conversion, and the period of repeating decimals in various bases.

The course makes very few assumptions about participants' prior knowledge of these topics. Because of this, the experience with specific groups of participants may be significantly different. This course was first delivered to a group of elementary and secondary teachers with very wide experience levels. With more experienced participants, expect Session 1 to last shorter than it might otherwise. With less experienced participants, note that Session 2 includes additional chances for them to gain skills in the prerequisite topics. Instead of direct instruction on modular arithmetic, ask participants to explain their reasoning in this session, and formalize during Sessions 2 and 3.

Notes on Selected Problems

Start by showing the video. Explain how the cards are being shuffled as clearly as possible, but watch out as participants are likely to make mistakes with their own shuffling routines.

Allow participants to work on the opener for an extended period (30 minutes or more), then direct their attention to Problem 1. Some participants are likely to ask that question for themselves, given that 52 cards is very large. Some participants will instead analyze suits and values; while this should not be explicitly discouraged, it is a fruitless direction, and Problem 1 is a better redirect. Watch for anyone following a specific card through the shuffles: this is a key concept that will become increasingly important in future sets. Related, some participants may try "grouping" cards that lead to one another. This will be a topic of interest later (see Session 4, Problem 7 for a start to this sequence).

Discourage anyone who begins talking in terms of mod arithmetic, asking them to rephrase their comments in a way that can be understood by everyone.

A participant may be able to explain at this point why the number of shuffles must be finite (each shuffle takes the deck to a new permutation, and there are only a finite number of them), but it is less likely participants can explain why the number of shuffles is always less than the deck size (and sometimes $n-2$ for n cards). If anyone has an explanation or proof for the 8 shuffles, save it for another day, since not everyone has had a chance to figure it out.

Problem 2 prepares some discussion around mod 10 that will happen in future sessions; in particular watch for participants giving only one answer for part (c) or getting upset about part (d), which has no solution.

In Problems 4, some participants did not like the "decimal" notation being used, in particular having trouble seeing the .2 in 2110.2_3 as two thirds. Working back over Problem 3b seemed to help.

The difference between Problems 6 and 7 is that some may write the fractions in Problem 6 directly as numerator and denominator; for example, writing 6e as $(1/111)_3$.

Problem 7 requires participants to write these answers as base-3 decimals. Problem 7c can be used to connect to a proof that 1 = 0.999... (Problem 13 in Session 2).

Problem Set 2

Goals of the Problem Set

One major goal of this session is to gather a large amount of data about the shuffling problem, in attempts to find and explain patterns in the data. Some simple patterns can be found, but other patterns are quite difficult. Encourage participants to make bold conjectures, since editing and correcting these conjectures is likely to lead to even better results. Participants may have a lot of trouble here breaking out of thinking about specific types of patterns (such as linear or quadratic) depending on their previous experiences in mathematics.

A second major goal is an introduction to modular arithmetic and the term *mod* m for 10 and 7. A long-term goal here is for participants to tease out important differences between mods; in particular, that a nonzero equation in mod 10 may have more than one solution while the same equation in mod 7 will always have one solution. Later, this will be tied to units and zero divisors, and all nonzero elements in a prime mod are units under multiplication.

Participants will also have opportunities to work a little more on base conversion. Since base conversion will be a major focus beginning in Session 5, these problems may reasonably be ignored or explored only by those who get through the rest of the problem set comfortably.

Notes on Selected Problems

Participants will need to decide how 9 cards should be shuffled before executing the opener. Be sure the participants are splitting the cards 5-and-4, and the rest should take care of itself. Watch for any participant deciding to “follow a card” through the shuffle; Problem 2 from Session 3 will ask the same question for an 8-card in-shuffle, which turns out to be equivalent to the 9-card shuffle seen here. The same applies if anyone followed a card from a 10-card out-shuffle in Session 1, but this is less likely to have been a focus.

In Problem 1, participants may prefer to build a shared spreadsheet (such as one through Google Drive) that allows for different people to enter their results from different deck sizes. This will give participants a large amount of information that can be used to verify data and look for patterns. The most obvious pattern here is that an odd number of cards returns to its original order in the same number of shuffles as the next-largest even number (9 and 10, for example). Participants may or may not be able to explain this; a good explanation is to imagine a phantom extra card at the bottom of the deck, where it would remain under the shuffling from Session 1.

Participants are likely to pursue patterns in the data from Problem 1 immediately. While fruitful, don't spend too much time like this or participants may not be able to get to Problem 5. Problem 6 is written to redirect participants to the pattern-finding task, but only after they have done the requisite work on modular arithmetic. This may also encourage participants to look for solutions involving modular arithmetic, although they are unlikely to find such a solution in this session.

Problems 2 and 3 encourage "mod 10" thinking before the definition in Problem 4. In particular, part (b) contains a zero units' digit in $(46 + 4)$ so the product is zero, and in part (c) all the fourth-powers end in 1. Part (c) is referenced again in Problem 4f.

Some participants will have trouble expressing the difference between mod 10 and base 10. Try bringing them back to the units digit problems (Problems 2 and 3) and help them to understand that $3 \cdot 4 = 2$ in this system; they are likely to say $3 \cdot 4 = 12$ and then want to do something to the 12. Explain that in mod 10, there is no such number as 12, but 2 "acts like" 12. More problems in Session 3 will address this issue, so don't worry too much if some of the confusion is unresolved.

Problem 7, if time permits, presents some connections to future work around remainders and decimal expansion. In particular, parts (c) through (f) are presented in order of the remainders as they would appear in long division: $10/7$ gives remainder 3, then $30/7$ gives remainder 2, then $20/7$ gives remainder 6. This is why the decimal expansions are "in sequence": $.\overline{142857}$, then $.\overline{428571}$, then $.\overline{285714}$, then $.\overline{857142}$.

CHAPTER

3 Solutions

Problem Set 1

Opener

Answers will vary. Some will try different deck sizes, others will look at specific cards moving through the positions in the deck. Eventually we'll prove that 8 shuffles works for a 52-card deck and find a rule that determines the number of shuffles for any deck, but that's a long way off . . .

1. No matter the deck size, the perfect shuffle appears to "work", returning the cards to their original positions. The number of required perfect shuffles varies, and it does not seem to have an obvious pattern; increasing the number of cards does not increase the number of shuffles needed.

When using an odd number of cards, a specific way to shuffle the deck must be used to consistently cut the deck into two parts, but either choice still leads to the same property.

2. a. Well, if we want 3 times a number to result in something ending in 4, the only option is if there is an 8 in the last position of that number. Thus, $x =$ __8

$$\begin{array}{r} \text{some number } x \\ \underline{\times \qquad\qquad\qquad 3} \\ \text{last digit of result is 4} \end{array}$$

b. If we want 7 times a number to result in something ending in 4, the only option is if there is a 2 in the last position of that number. Thus, $x = \_\_2$.

c. If we want 4 times a number to result in something ending in 4, then we have a couple of choices here. We can get a 4 if there is a 1 or 6 in the last digit of x. So $x = \_\_1$ or $x = \_\_6$.

d. 5 times any number results in a number ending in a 5 or 0, so it is not possible to have a resulting number where the last digit is a 4.

3. **a.** $9 \cdot 10^1 + 9 \cdot 10^0 + 4 \cdot 10^{-1} + 4 \cdot 10^{-2} = 90 + 9 + \frac{4}{10} + \frac{4}{100} = 90 + 9 + 0.4 + 0.04 = 99.44$

b. $802.11_{10} = 800 + 0 + 2 + 0.1 + 0.01 = 8 \cdot 100 + 0 \cdot 10 + 2 \cdot 1 + 1 \cdot \frac{1}{10} + 1 \cdot \frac{1}{100} = 8 \cdot 10^2 + 0 \cdot 10^1 + 2 \cdot 10^0 + 1 \cdot 10^{-1} + 1 \cdot 10^{-2}$

4. **a.** $1 \cdot 3^3 + 0 \cdot 3^2 + 2 \cdot 3^1 + 0 \cdot 3^0 + 1 \cdot 3^{-1} = 27 + 0 + 6 + 0 + \frac{1}{3} = 33\frac{1}{3} = 33.\overline{3}$ in base 10. We would write it in base 3 as: 1020.1_3

b. $2110.2_3 = 2 \cdot 3^3 + 1 \cdot 3^2 + 1 \cdot 3^1 + 0 \cdot 3^0 + 2 \cdot 3^{-1}$

c. As a base 10 number, $2110.2_3 = 54 + 9 + 3 + 0 + \frac{2}{3} = 66\frac{2}{3} = 66.\overline{6}$

5. **a.** 0.5

b. 0.02

c. $0.\overline{1}$

d. $0.\overline{2}$

e. 1.0

f. $0.\overline{076923}$

6. **a.** $9 = 3^2 = 1 \cdot 3^2 = 1 \cdot 3^2 + 0 \cdot 3^1 + 0 \cdot 3^0 = 100_3$

b. The largest power of 3 without going over is...2! But 3^2 will only get us 9 and we have 4 more to go. Well, 3 is less than 4...then we just have 1 left!
$13 = 9 + 3 + 1 = 1 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 = 111_3$

c. Now things are getting big! Okay, $3^4 = 81$ and $3^5 = 243$. So 3^5 is too big, so how many 3^4's can we fit into 242 without going over?
$2 \cdot 3^4 = 162$ (note that $3 \cdot 3^4 = 3^5$, so no can do).
This will leave $242 - 162 = 80$.
Okay, how many 3^3's can we fit into 80 without going over? Two of them, since $2 \cdot 3^3 = 54$, leaving us with 26 to deal with...
How many 3^2's can we fit into 26? Two of them, which leaves 8. Two 3^1's will fit into 8, and then we can account for the remaining 2 with two 3^0's.

Summary: $242 = 2{\cdot}3^4+2{\cdot}3^3+2{\cdot}3^2+2{\cdot}3^1+2{\cdot}3^0 = 22222_3$

d. $\dfrac{1}{9} = \dfrac{1}{3^2} = 1 \cdot 3^{-2} = 0.01_3$

e. How many $\dfrac{1}{9}$'s fit into $\dfrac{1}{13}$? Wait! That's too big! Hmmm... now we have to look at things a little differently. How many $\dfrac{1}{27}$'s fit into $\dfrac{1}{13}$? Two, with $\dfrac{1}{351}$ left over. We know we cannot use three $\dfrac{1}{27}$'s, we would go over. And again with this process. There has got to be a better way!

Let's convert the numerator and denominator of $\dfrac{1}{13}$ to base 3 numbers:

$\dfrac{1}{13} = \dfrac{1_3}{111_3}$

How about some base 3 long division!

$$\begin{array}{r} 0.002002_3 \\ 111_3\overline{)1.000000_3} \\ \underline{-222_3} \\ 1000 \\ \underline{-222_3} \\ 1 \end{array}$$

A repeated remainder!

So, $\dfrac{1}{13} = 0.\overline{002}_3$.

f. Let's stick with the long division way. That seems easiest.

$\dfrac{1}{242} = \dfrac{1_3}{22222_3}$

Let the long division commence!

$$\begin{array}{r} 0.00001_3 \\ 22222_3\overline{)1.00000_3} \\ \underline{-22222_3} \\ 1_3 \end{array}$$

Repeat!

$\dfrac{1}{242} = 0.\overline{00001}_3$

7. **a.** 0.01_3

b. $\dfrac{1}{2} = \dfrac{1_3}{2_3}$ Ha!

$$\begin{array}{r} 0.1_3 \\ 2_3\overline{)1.0_3} \\ -\underline{2_3} \\ 1_3 \end{array}$$

Repeat!
So $\frac{1}{2} = 0.\overline{1}_3$

c. $\frac{2}{2} = 1 = 3^0 = 1 \cdot 3^0 = 1_3$

d. $\frac{3}{2} = 1 + \frac{1}{2}$, so let's use what we got in parts b and c.
$\frac{3}{2} = 1.\overline{1}_3$

e. $0.\overline{001}_3$

f. $0.\overline{00001}_3$

8. Since terminating decimals are in the form of tenths, hundredths, thousandths, and so on, the denominator of the *completely reduced* fraction must *not* contain any number other than 2 or 5 in its prime factorization. So if the completely reduced contains any number other than 2 or 5, then the decimal will repeat in base-10.

9. Looking back at the repeating decimals from problems 6 and 7, it appears the ones that terminate all have denominators that have a prime factorization containing only 3's.

10. **a.** $\dfrac{1}{13}$ (see #6e)

b. $\dfrac{2}{63}$

c. $\dfrac{1}{62}$

d. $\dfrac{1}{171}$

e. Well, these seem to be following the pattern of $\dfrac{2}{n^3 - 1}$

f. Following the pattern this would appear to equal $\dfrac{2}{7}$, but requires some extra work to explain, as the "2" would move to a different position: the decimal would then be equivalent to $\overline{.010}_2$, which works!

Problem Set 2

Opener

Yes, it took 6 shuffles. Hey! That's the same number that it took for 10 cards. Interesting . . .

1. Complete this table with the number of perfect shuffles needed to restore a deck to its original state.

n	# shuffles
3	2
4	2
5	4
6	4
7	3
8	3
9	6
10	6
11	10
12	10
13	12
14	12
15	4
16	4
17	8
18	8

n	# shuffles
19	18
20	18
21	6
22	6
23	11
24	11
25	20
26	20
27	18
28	18
29	28
30	28
31	5
32	5
33	10
34	10

2. The units digit, so I only care about the unit digits of the numbers...

- **a.** $4 \cdot 6 + 3 \cdot 4 =$ ... only care about the units, so $4 + 2 = 6$
- **b.** Hey! $46 + 4$ has a 0 in the units digit, so the result will have 0 in the units digit. Blam!
- **c.** All of these numbers have a 1 in the units digit, so the result will as well.

3.
- **a.** units digit $= 7$
- **b.** units digit $= 9$
- **c.** units digit $= 4$ or 9
- **d.** Not possible! Numbers that are multiplied by 5 end in a 5 or 0. David's off his rocker!

4.
- **a.** $2 + 2 = \boxed{4}$
- **b.** $3 \cdot 4 = \boxed{2}$
- **c.** $\boxed{7} + 5 = 2$
- **d.** $4 \cdot \boxed{3} = 2$, or $4 \cdot \boxed{8} = 2$
- **e.** $5 \cdot \boxed{?} = 3$ Not possible, since the remainder would be 0 or 5.
- **f.** $\boxed{1}^4 = 1$, $\boxed{3}^4 = 1$, $\boxed{7}^4 = 1$, $\boxed{9}^4 = 1$

5. **a.** $2 + 2 = \boxed{4}$

b. $3 \cdot 4 = \boxed{5}$

c. $\boxed{4} + 5 = 2$

d. $4 \cdot \boxed{4} = 2$

e. $5 \cdot \boxed{2} = 3$

f. $\boxed{1}^4 = 1, \boxed{6}^4 = 1$

6. Decks that have 2^n cards take n shuffles, and decks that have $2^n + 2$ cards take $2n$ shuffles. The most obvious pattern is that odd-size decks take as many shuffles as the next even number.

7. **a.** $\frac{1}{5} = 0.2$

b. $\frac{1}{25} = 0.04$

c. $\frac{1}{7} = 0.\overline{142857}$

d. $\frac{3}{7} = 0.\overline{428571}$

e. $\frac{2}{7} = 0.\overline{285714}$

f. $\frac{6}{7} = 0.\overline{857142}$ What's going on here?!

g. $\frac{1}{13} = 0.\overline{076923}$

h. $\frac{2}{13} = 0.\overline{153846}$

8. **a.** $\frac{1}{13} = 0.\overline{002}_3$

b. $\frac{2}{13} = 2 \cdot \frac{1}{13} = 0.\overline{011}_3$

c. $\frac{3}{13} = 3 \cdot \frac{1}{13} = 0.\overline{020}_3$

d. $\frac{9}{13} = 0.\overline{200}_3$

e. $\frac{10}{13} = 0.\overline{002}_3 + 0.\overline{200}_3 = 0.\overline{202}_3$

f. $\frac{1}{7} = 0.\overline{010212}_3$

g. $\frac{3}{7} = 3 \cdot \frac{1}{7} = 0.\overline{102120}_3$

h. $\frac{9}{7} = 1 + \frac{2}{7} = 1.\overline{021201}_3$

i. $\frac{6}{7} = 0.\overline{212010}_3$

9. $\frac{1}{142857} = .\overline{000007}$