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The Mathematics of Soap Films: Explorations with Maple®

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Preface

Scientific principles are often reflected in geometry. Whether it is the curve made by a hanging wire or the path that light takes around the sun, shapes are often the manifestations of Nature's design. This book is about the mathematics which describes the geometric properties of soap films. Using ideas from plane geometry, differential geometry, complex analysis and the calculus of variations, we can begin to understand why soap films take the shapes they do. But it isn't just soap which interests us as mathematicians. Rather, it is the mathematization of the study of soap film shapes which serves as a prime example of the place geometry has in mathematical modeling.

As we shall see, the effects of surface tension lead a soap film to minimize its surface area. This well-defined mathematical consequence allows us to study soap film shapes from a purely mathematical viewpoint. The mathematics involved ranges from the elementary to the very advanced, but in this book we will focus on a point somewhere in the middle. That is, readers are expected to know calculus and have some familiarity with differential equations, but the relevant notions from differential geometry and complex variables needed in the book are all discussed in Chapter 2. In fact, in order to get to the point quickly, we try to use only the essential ingredients of each of these subjects to begin to tell the mathematical story of soap films.

With this in mind, the book could serve as a text for a junior-senior level seminar course or an independent study course.

Basic references which vastly extend the exposition presented here are [**Nit89**], [**DHKW92**], [**Oss86**], [**Boy59**], [**Mor88**], [**Tho92**], [**HT85**] and [**Ise92**]. In particular, [**Nit89**] and [**HT85**] provide histories and overviews of the study of minimal surfaces (i.e. mathematical soap films) and the uses of minimal surfaces in science, engineering and architecture, while [**Mor88**] presents an introduction to geometric measure theory (the *modern* approach to minimal surfaces) which every budding geometer should read. For a straightforward discussion of surface tension and its effects, [**Ise92**] can't be beat.

In the past, minimal surfaces have been considered in differential geometry books as an add-on ([**Gra93**], [**Opr97**]) and in other books at perhaps too high a level for undergraduates. Also, the basics of surface tension have not been discussed in these books, but left to higher-level texts ([**Fin86**]) or books tending more toward the physics side of things ([**Ise92**]). So a major goal of this book is simply to make available a self-contained text where undergraduates can see a mixture of the different types of mathematics which pertain to minimal surfaces together with a bit of the science behind the subject. Most undergraduates only see the standard applications of mathematics to physical systems, where answers are typically only analytic. Here they can see another way that mathematics applies to science — the determination of optimal shape.

Since this book concentrates on shape, it would be a shame not to present the reader with ways of creating shapes. With this in mind, almost 40% of the book is devoted to exploring various notions using the software package Maple. So, in the last third of the book, the reader will 'see' fluids rising up inclined planes, create minimal surfaces from complex variable data and investigate the 'true' shape of a balloon. In fact, rather than reading the book in order, the reader is recommended to jump back and forth from the mathematical exposition to the relevant Maple work. It really pays nowdays for undergraduates to learn a package such as Maple or Mathematica, so the Maple work given is usually discussed at length. There is much to be learned in making computers actually do *interesting* things in mathematics.

With this in mind, I would like to thank John Reinmann for his expertise and interest in Maple applied to soap films, bubbles and, especially, variational problems. John's help has been invaluable. Also, my daughter Kathy has been my assistant in soap film demonstrations over the years and I would like to tell her here how much I appreciate that. In fact, the photographs of soap films in Chapter 1 were taken by Kathy Oprea and Katie Cline. These demonstrations and photographs are the outgrowth of a grant from Cleveland State University which enabled me to commission the sculptor Ron Dewey to create the wireframes needed for experimentation. Thanks to CSU and to Ron for his fine work. Finally, I'd like to thank my wife Jan for her support and for giving me my own small part of the house for my wireframes, a bucket of ever-ready soap solution and all the rest of my toys.

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