
Preface to the Second Edition and Acknowledgment

Chapter 10 on an introduction to sieve methods and additive number theory is added. I originally intended to separate these two topics, but for simplicity, I combined them into a single chapter. This is a new chapter that does not appear in the first edition.

Some typographical and grammatical errors are corrected. The notes at the end of each chapter are updated. English and latex sentences are slightly changed to remove some bad boxes, which occur due to the change of latex style from the first to the second edition. Although I tried to remove as many of them as possible, some bad boxes are left unchanged. I apologize for any inconvenience.

I hope the reader enjoys reading this book, and if you have any comments or suggestions, please send me an email at prapanpong@gmail.com or pongsriiam_p@silpakorn.edu. I will also put the list of updates on my google homepage at <https://sites.google.com/view/math-prapanpong-pongsriiam/teaching>.

Many people have supported me, directly or indirectly, during the preparation of this book. Some of their names already appeared in the first edition, but there are others I would like to mention too.

First of all, I would like to thank the editors Eriko Hironaka and Ina Mette for their patience and understanding during the revision and publication process. The reviewers gave me an insight and made me aware of some references that I could use to update my notes. They also suggested that I write a new chapter on sieve methods and additive number theory, and recommended that I add more prose and connecting sentences. I thank all three anonymous reviewers for their kind comments and useful suggestions which improved the quality of the manuscript. I will be glad if this book meets parts of their expectations.

Associate Professor Wanida Hemakul supported me in the application of the Tosio Kato Fellowship of the Mathematical Society of Japan. Professor Kohji Matsumoto agreed to be my host professor and let me do research at Nagoya University. The draft of the second edition was first written while I was working in Thailand, but the final version was finished when I visited Nagoya University in Japan. The Library of Nagoya University let me have access to several books, articles, references, and the important database MathScinet™, which I did not have access to while I was in Thailand. Yuko Ikuta and her colleagues helped me with some of the paper work. I am thankful to all of them.

Most importantly, I am very grateful to Tammatada Khemaratchatakumthorn for helping me prepare the entire draft of both the first and second editions.

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Preface to the First Edition

I took several mathematics courses when I was a graduate student at Penn State University. With Professor Vaughan's teaching and the book by Professors Montgomery and Vaughan as references, I decided to write a lecture note on analytic number theory which was accessible to beginners.

My small lecture note has gradually been expanded from various courses, seminars, and conference talks I have given at Silpakorn University and many other institutions. It has been distributed among the audiences ranging from advanced high school to doctoral students, and it has finally become this text. While most textbooks on the same subject are written for advanced graduate students or researchers, this book contains the material for both undergraduate and graduate students, and also offers a transition between them.

Chapters 1 to 3 have been used in undergraduate teaching and also in the training of selected students at IPST for International Mathematical Olympiad. Chapters 4 to 9 are for graduate students and any reader who would like a taste of analytic number theory. The prerequisites for this book are undergraduate courses in elementary number theory, real analysis, complex analysis, and abstract algebra. Beyond this, the volume is self-contained. I show the details of the proofs as much as possible so that the readers can follow them easily. However, some parts may

be more difficult than the others, and therefore it is optional to study or skip them in the first reading. This option is clearly indicated in the second part given below.

Examples and exercises are of varying difficulty and are collected from classic texts, mathematical competitions, and my personal notes taken when I attended Professor Vaughan's lecture; some are from my design; some are from joint work with my students; many of these are connected with possible research projects for students and researchers. I have also tried to give credit to the sources of each exercise. However, I may miss something. If the readers find anything inaccurate, I will be happy to learn more and will adjust my book accordingly.

At the end of each chapter, I intentionally record many things in the notes including a summary of the lessons, a source and a connection of exercises with open questions, a short history of an invention in mathematics, an entertaining fact, my results that are related to the chapter, and my thoughts on some open problems. Unlike many other books on the same subject, I point to various directions in the hope that the readers may get new ideas not only on the central topic but also totally different topics of research. I think this gives a broadly accessible survey and a wider view of analytic-combinatorial-elementary number theory, and I hope this provides the interested reader with ample material to pursue.

It will be a pleasure too if the readers solve some of my proposed problems or any other related research question, get them published, and let me know the answer. Please send any comments to my email at prapanpong@gmail.com or pongsriiam_p@silpakorn.edu. I will also post a list of errata and other updates on my homepage in the future.

How to Use This Book. Chapter 1 (except the notes) is just a review of elementary number theory. So the reader may read through it quickly. Chapter 2 is also known to undergraduate students, but our presentation is quite different, and studying it will help the reader get familiar with the basic calculations in later chapters. Chapter 3 on the floor function is interesting on its own and can be used as a supplementary reading for undergraduates, but reading only Definition 3.1 and Theorem 3.3 is more than enough for understanding the other chapters. Chapter 4 gives the basic notation and summation formulas that are used throughout this text. Chapters 5 to 9 contain the main material for the first course

in analytic number theory. However, Sections 4.7, 5.4, 5.5, and 9.3 to 9.9 may be skipped in the first reading.

It is not necessary to solve all the exercises. Although many theorems in number theory are very difficult to prove, understanding their statements is easy. So whenever we refer to a result in an exercise, the readers can simply assume that it is true, use it in any way they like, and then come back at a time later to solve it. In fact, exercises are often cited in the notes but only one theorem (Theorem 5.7) depends on an exercise.

Notes at the end of each chapter are meant to entertain and inform the readers on old and new results in mathematics; they are not used in the proofs of any theorems in this book, and so the readers do not have to study all of them. Although the notes may depend heavily on my preferences, they contain the discussion of research that is related to the main material and exercises. Some research problems may be suitable for students and I hope that experts and instructors can also use these in their teaching. Perhaps my favorite topics are in the notes of Chapters 1, 2, 3, 4, 5, 7 and 9. I hope the readers will like them too. Again, I welcome any suggestions from the readers.

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in collecting numerical data for my research, which in turn supported me in designing some exercises and expanding some notes.

The Faculty of Science at Silpakorn University has given me financial support¹ and had my manuscript reviewed by three external reviewers whose comments and suggestions helped to improve the quality of this book.

The editors and reviewers of the American Mathematical Society (AMS) also reviewed my manuscript and gave recommendations. I would like to thank them for their generosity for allowing me to distribute the first edition of the book before adding two or three more chapters and publishing the new version with the AMS as this second edition. By the way, the topics to be added are on sieve methods, additive number theory, and the circle method.

I have several opportunities to join and give talks in the seminars organized by Aram Tangboonduangjit, Thotsaporn Thanatipanonda, Chatchawan Panraksa, and Pornrat Ruengrot at Mahidol University International College (MUIC). Their hospitality and the academic atmosphere at MUIC remind me of the good old days when I was listening to Professor Vaughan's lectures and attending seminars at Penn State and other institutions. In addition, I thank Professor Vichian Laohakosol and his students for organizing the CNA and ICNA conferences every year, which promotes the research activity among my students and other number theorists in Thailand.

Special thanks go to Nobuo Uematsu, Yasunori Mitsuda, Kenji Ito, Yoko Shimomura, Hiroki Kikuta, and other musicians and artists at Square Soft (now Square Enix) for making the video games enjoyable and beautiful. Listening to their music not only brings back the exciting adventures and good memories of my childhood but also gives me a lot of energy to work on mathematics.

Last but not least, I am very grateful to Tammatada Khemaratchatakumthorn who helped me prepare the entire manuscript and has been supporting me in all other aspects of my life. Without her, this text would never have appeared. Any good coming from this is dedicated to her.

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