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Volume 11

An Introduction to Game-Theoretic Modelling

Second Edition

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Preface

This is the second edition of a short introduction to game theory and applications from the perspective of a mathematical modeller, and includes several original pedagogical games. It covers a range of concepts that have proven useful, or are likely to prove useful, for quantitative modelling in the life, social and management sciences. Its approach is heuristic, but systematic, and it deals in a unified manner with the central ideas of both classical and evolutionary game theory. In many ways, it is a sequel to my earlier work, *A Concrete Approach to Mathematical Modelling* [**144**],[1] in which games were not discussed. The mathematical prerequisites are correspondingly modest: calculus, a rudimentary knowledge of matrix algebra and probability, a passing acquaintance with differential equations and that intangible quantity, a degree of mathematical maturity. Naturally, the greater one's maturity, the more contemptuous one can be of formal prerequisites, and the more one is able to proceed ad hoc when the need arises.

Reactions to the first edition of this book have been positive and pleasing. Although I haven't fixed what isn't broken, I have paid attention to feedback: I have added a brand new chapter on population games, I have included some new examples and exercises, I have corrected all known errors (and tried hard not to introduce new

[1] Bold numbers in square brackets are references listed on pp. 347-361.

ones), and I have brought the references up to date. By design, the references are selective: it would conflict with the goals of this series to cite all potentially relevant published literature. Nevertheless, I have tried to make the vast majority of it easily traceable through judicious references to more recent work.[2]

There exist several excellent texts on game theory, but their excellence is largely for the mathematical purist. Practices that are de rigueur to the purist are often merely distracting to the modeller—for example, lingering over the elegant theory of zero-sum games (nonzero-sum conflicts are much more common in practice), or proving the existence of a Nash equilibrium in bimatrix games (for which the problem in practice is usually to distinguish among a superabundance of such equilibria); or, more fundamentally, beginning with the most general possible formulation of a game and only later proceeding to specific examples (the essence of modelling is rather the opposite). Such practices are therefore honored in the breach. Instead—and as described more fully in the agenda that follows—the emphasis is on concrete examples, and the direction of pedagogy throughout the book is from specific to general. However bright or well motivated, students often have limited appetites for rigor and generality, yet have much still to gain from the mathematical experience of capturing ideas and giving them substance—the experience, that is, of modelling. I hope that such students will find that this book helps not only to make game theory accessible, but also to convey both its power and scope in a variety of applications.

[2] And for the very latest, there exists the internet; see [**136**, pp. 147-149] for links.