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Problems in Mathematical Analysis II

Continuity and Differentiation

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Preface

This is the second volume of a planned series of books of problems in mathematical analysis. The book deals with real functions of one real variable, except for Section 1.7 where functions in metric spaces are discussed. Like the first volume, *Problems in Mathematical Analysis I, Real Numbers, Sequences and Series*, the book is divided into two parts. The first part is a collection of exercises and problems, and the second contains their solutions. Although often various solutions of a given problem are possible, we present here only one. Moreover, problems are divided into sections according to the methods of their solutions. For example, if a problem is in the section *Convex Functions* it means that in its solution properties of convex functions are used. While each section begins with relatively simple exercises, one can still find quite challenging problems, some of which are actually theorems. Although the book is intended mainly for mathematics students, it covers material that can be incorporated by teachers into their lectures or be used for seminar discussions. For example, following Steven Roman (Amer. Math. Monthly, 87 (1980), pp. 805-809), we present a proof of the well known Faà di Bruno formula for the nth derivative of the composition of two functions. Applications of this formula to real analytic functions given in Chapter 3 are mainly borrowed from the book *A Primer of Real Analytic Functions* by Steven G. Krantz and Harold R. Parks. In fact, we found this book so stimulating that we could not resist borrowing a few theorems from it. We would like also to mention here a generalization of Tauber's theorem due to Hardy and Littlewood. The proof of this result that we give is based on Karamata's paper (Math. Zeitschrift, 2 (1918)).

Many problems have been borrowed freely from problem sections of journals like the American Mathematical Monthly, Mathematics Today (Russian) and Delta (Polish), and from many textbooks and problem books. The complete list of books is given in the bibliography. As in the first volume, it was beyond our scope to trace all original sources, and we offer our sincere apologies if we have overlooked some contributions.

All the notations and definitions used in this volume are standard. However, in an effort to make the book consistent and to avoid ambiguity, we have included a list of notations and definitions. In making references we write, for example, 1.2.33 or I, 1.2.33, which denotes the number of the problem in this volume or in Volume I, respectively.

We owe much to many friends and colleagues with whom we have had many fruitful discussions. Special mention should be made, however, of Tadeusz Kuczumow for suggestions of several problems and solutions, and of Witold Rzymowski for making his manuscript [28] available to us. We are very grateful to Armen Grigoryan, Małgorzata Koter-Mórgowska, Stanisław Prus and Jadwiga Zygmunt for drawing the figures and for their help with incorporating them into the text. We are deeply indebted to Professor Richard J. Libera, University of Delaware, for his unceasing help with the English translation and for his valuable suggestions and corrections which we feel have greatly improved both the form and the content of the two volumes. Finally, we would like to thank the staff at the AMS for their dedicated assistance (via e-mail) in bringing our work to fruition.

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