Preface

Mathematical billiards describe the motion of a mass point in a domain with elastic reflections from the boundary. Billiards is not a single mathematical theory; to quote from [**57**], it is rather a mathematician's playground where various methods and approaches are tested and honed. Billiards is indeed a very popular subject: in January of 2005, MathSciNet gave more than 1,400 entries for "billiards" anywhere in the database. The number of physical papers devoted to billiards could easily be equally substantial.

Usually billiards are studied in the framework of the theory of dynamical systems. This book emphasizes connections to geometry and to physics, and billiards are treated here in their relation with geometrical optics. In particular, the book contains about 100 figures. There are a number of surveys devoted to mathematical billiards, from popular to technically involved: [**41, 43, 46, 57, 62, 65, 107**].

My interest in mathematical billiards started when, as a freshman, I was reading [**102**], whose first Russian edition (1973) contained eight pages devoted to billiards. I hope the present book will attract undergraduate and graduate students to this beautiful and rich subject; at least, I tried to write a book that I would enjoy reading as an undergraduate.

This book can serve as a basis for an advanced undergraduate or a graduate topics course. There is more material here than can be realistically covered in one semester, so the instructor who wishes to use the book will have enough flexibility. The book stemmed from an intense[1] summer REU (Research Experience for Undergraduates) course I taught at Penn State in 2004. Some material was also used in the MASS (Mathematics Advanced Study Semesters) Seminar at Penn State in 2000–2004 and at the Canada/USA Binational Mathematical Camp Program in 2001. In the fall semester of 2005, this material will be used again for a MASS course in geometry.

A few words about the pedagogical philosophy of this book. Even the reader without a solid mathematical basis of real analysis, differential geometry, topology, etc., will benefit from the book (it goes without saying, such knowledge would be helpful). Concepts from these fields are freely used when needed, and the reader should extensively rely on his mathematical common sense.

For example, the reader who does not feel comfortable with the notion of a smooth manifold should substitute a smooth surface in space, the one who is not familiar with the general definition of a differential form should use the one from the first course of calculus ("an expression of the form..."), and the reader who does not yet know Fourier series should consider trigonometric polynomials instead. Thus what I have in mind is the learning pattern of a beginner attending an advanced research seminar: one takes a rapid route to the frontier of current research, deferring a more systematic and "linear" study of the foundations until later.

A specific feature of this book is a substantial number of digressions; they have their own titles and their ends are marked by ♣. Many of the digressions concern topics that even an advanced undergraduate student is not likely to encounter but, I believe, a well educated mathematician should be familiar with. Some of these topics used to be part of the standard curriculum (for example, evolutes and involutes, or configuration theorems of projective geometry), others are scattered in textbooks (such as distribution of first digits in various sequences, or a mathematical theory of rainbows, or the 4-vertex theorem), still others belong to advanced topics courses (Morse theory, or Poincaré recurrence theorem, or symplectic reduction) or

[1]Six weeks, six hours a week.

simply do not fit into any standard course and "fall between cracks in the floor" (for example, Hilbert's 4-th problem).

In some cases, more than one proof to get the same result is offered; I believe in the maxim that it is more instructive to give different proofs to the same result than the same proof to get different results. Much attention is paid to examples: the best way to understand a general concept is to study, in detail, the first non-trivial example.

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Serge Tabachnikov