
Contents

Introduction	1
Part 1. Recollections	
Chapter 1. Linear Representations of Finite Groups	9
§1.1. Groups	9
§1.2. Homomorphisms of Groups	14
§1.3. Linear Representation of Groups	18
§1.4. Exercises	24
Chapter 2. Rings and Algebras	27
§2.1. Rings	27
§2.2. Homomorphisms of Rings, Algebras	31
§2.3. Ideals	33
§2.4. Exercises	40
Part 2. Introduction and Göbel's Bound	
Chapter 3. Rings of Polynomial Invariants	45
§3.1. Linear Group Actions	45
§3.2. Rings of Invariants	53
§3.3. Exercises	60

Chapter 4. Permutation Representations	63
§4.1. Permutation Representations	63
§4.2. Newton, Waring, and Gauss	67
§4.3. Göbel's Bound	75
§4.4. Exercises	87
Application: Decay of a Spinless Particle	91
Application: Counting Weighted Graphs	95
 Part 3. The First Fundamental Theorem of Invariant Theory and Noether's Bound	
Chapter 5. Construction of Invariants	99
§5.1. Orbit Chern Classes	99
§5.2. The Transfer	103
§5.3. New Invariants from Old Ones	109
§5.4. Exercises	113
Chapter 6. Noether's Bound	117
§6.1. The Noether Map	117
§6.2. Polarizations	123
§6.3. The First Fundamental Theorem of Invariant Theory for Σ_d	127
§6.4. Noether's Bound	130
§6.5. Exercises	135
Chapter 7. Some Families of Invariants	137
§7.1. Invariants of Representations of Cyclic Groups	137
§7.2. Pseudoreflection Groups	141
§7.3. Vector Invariants	148
§7.4. Exercises	151
Application: Production of Fibre Composites	155
Application: Gaussian Quadrature	159

Part 4. Noether's Theorems

Chapter 8. Modules	165
§8.1. Modules and Module Homomorphisms	165
§8.2. Operations on Modules	170
§8.3. Direct Sums, Free Modules, and Finite Generatedness	172
§8.4. Maschke's Theorem and Schur's Lemma	176
§8.5. Exercises	182
Chapter 9. Integral Dependence and the Krull Relations	185
§9.1. Integral Dependence	185
§9.2. Integral Extensions	190
§9.3. The Krull Relations	193
§9.4. Exercises	198
Chapter 10. Noether's Theorems	201
§10.1. Finite Generatedness	201
§10.2. The Krull Dimension	208
§10.3. Examples	212
§10.4. Exercises	217
Application: Self-Dual Codes	219

Part 5. Advanced Counting Methods and the Shephard-Todd-Chevalley Theorem

Chapter 11. Poincaré Series	225
§11.1. Poincaré Series	225
§11.2. Molien's Theorem	233
§11.3. Exercises	242
Chapter 12. Systems of Parameters	245
§12.1. System of Parameters	245
§12.2. Hilbert's Nullstellensatz	249
§12.3. Primary and Secondary Generators	252

§12.4. Dade's Basis	258
§12.5. The Degree Theorem	261
§12.6. Exercises	265
Chapter 13. Pseudoreflexion Representations	267
§13.1. Pseudoreflections	267
§13.2. The Shephard-Todd-Chevalley Theorem, Part I	270
§13.3. The Shephard-Todd-Chevalley Theorem, Part II	275
§13.4. Exercises	281
Application: Counting Partitions	283
Appendix A. Rational Invariants	287
§A.1. Algebraic Field Extensions	287
§A.2. Splitting Fields and Normal Extensions	293
§A.3. Galois Extensions	295
§A.4. Examples	298
§A.5. Exercises	302
Suggestions for Further Reading	303
Notation Index	305
Subject Index	307