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Problems in Mathematical Analysis I

Real Numbers, Sequences and Series

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Preface

This book is an enlarged and revised English edition of a Polish version published in 1996 by the Publishing House of Maria Curie-Skłodowska University in Lublin, Poland. It is the first volume of a planned series of books of problems in mathematical analysis. The second volume, already published in Polish, is under translation into English. The series is mainly intended for students who take courses in basic principles of analysis. The choice and arrangement of the material make it suitable for self-study, and instructors may find it useful as an aid in organizing tutorials and seminars.

This volume covers three topics: real numbers, sequences, and series. It does not contain problems concerning metric and topological spaces, which we intend to present in subsequent volumes.

The book is divided into two parts. The first part is a collection of exercises and problems, and the second contains their solutions. Complete solutions are given in most cases. Where no difficulties could be expected or when an analogous problem has already been solved, only a hint or simply an answer is given. Very often various solutions of a given problem are possible; we present here only one, hoping students themselves will find others.

With the student in mind, we have tried to keep things at an elementary level whenever possible. For example, we present an elementary proof of the Toeplitz theorem about the so-called regular transformation of sequences, which in many texts is proved by methods of functional analysis. The proof presented is taken from Toeplitz's original paper, published in 1911 in Prace Matematyczno-Fizyczne, Vol. 22. We hope that our presentation of this part of real analysis will be more accessible to readers and will ensure wider understanding.

All the notations and definitions used in this volume are standard and commonly used. The reader can find them, for example, in the textbooks [12] and [23], in which all necessary theoretical background can be found. However, to make the book consistent and to avoid ambiguity, a list of notations and definitions is included.

We have borrowed freely from many textbooks, problem books and problem sections of journals like the American Mathematical Monthly, Mathematics Today (Russian) and Delta (Polish). A complete list is given in the bibliography. It was beyond the authors' scope to trace all original sources, and we may have overlooked some contributions. If this has happened, we offer our sincere apologies.

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