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The Game's Afoot! Game Theory in Myth and Paradox

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American Mathematical Society

Contents

Foreword

> Enigmatic for the masses,
> playfully with life we fool.
> That which human wits surpasses
> draws our special ridicule.[1]
> —**Christian Morgenstern,** *Gallows Hill*

No other mathematical discipline has altered the study of economics, the social sciences, and biology as has game theory, in the fifty years since its inception. Social traps, political mock battles, evolutionary confrontations, economic struggles, and not least literary conflicts can all be viewed as "games" of this theory.

This book is addressed to readers who are prepared to consider the perspective of a study, both formal and with practical application, that embraces science, literature, and life's conflicts.

For the layperson unencumbered by any previous knowledge of game theory, an introduction to the subject does not require graduate study in higher mathematics. An ability to think logically that does not shrink from entertaining sophistry will do nicely for passing unscathed through the hall of mirrors of strategic decision-making.

With the help of formulas, fables, and paradoxes we shall begin our lighthearted excursion into the world of strategic calculation. The

[1] Call it infantile vendetta/ on life's deeply serious aim—/ you will know existence better/ once you understand our game. [Translation by Max Knight.]

stations of this journey support the mathematics of conflict, and provide a connecting thread through the labyrinth of solution concepts and the unraveling of the myths of game theory. Our fanciful introduction to contemporary mathematical game theory stretches from the dilemma of the arms race by way of disaster on the internet to a lesson in the just division of a cake.

If there is a model for this undertaking, then it must be the book that made accessible to me—during my far-off student days—the notions of game, strategy, and saddle point, namely, J.D. Williams's *The Compleat Strategyst: Being a Primer on the Theory of Games of Strategy* [**95**].

Every refreshing inclination that winked at me from this collection incited my appetite for game-theoretic excursions in literary realms. I invite the reader who is so disposed to follow me on this path to an appreciation of game theory.

Chapter 3

In the Forest of Game Trees

A fool sees not the same tree that a wise man sees.
—**William Blake,** *The Marriage of Heaven and Hell*

The first game trees stretched forth their leafy branches in von Neuman and Morgenstern's monograph [**71**]. Kuhn's [**54**] concept of strategies for these complicated positional games was rather simple: a function specifying the player's action in each of his information sets.

While it is always possible to bring the formulation of a game tree into abstract normal form in order to carry out a successful search for equilibria, mixed equilibria in normal form provide no immediately understandable pattern of behavior in extensive alternate-move games.

It was again Kuhn who showed the way out of this dilemma. In [**54**] he showed that for the class of extensive games in which all players are characterized by perfect memory[1] there exists an equivalent way of representing mixed strategies. For every information set in which it is his move, a *behavioral strategy* specifies the probability for choosing each of the player's available actions (a probability distribution over the player's available actions).

[1] As regards their past knowldege and moves they have already made.

In this chapter we shall explain the strategic properties of extensive modes of play with the help of examples of game situations. The credibility of strategic equilibria will be put under the microscope in the first of these games. We have Selten [**84, 85**] to thank for the insight that many equilibria in unreachable parts of the game tree yield questionable, since nonequilibrium, recommendations.

In what follows we shall turn our attention to the bestiary of game theory. Rosenthal's "centipede," Selten's "horse," as well as Kohlberg's "dalek" illuminate many of the ideas of game theory, such as those of further refinement theorems and backward and forward induction.

3.1. The Strange Case of Lord Strange

> He said, "giue me my battell axe in my hand,
> sett the crowne of England on my head soe hye!
> ffor by him that shope both sea and Land,
> King of England this day I will dye!"
> — *Ballad of Bosworth Field*

He was awakened in the morning twilight from a fitful sleep. Shivering, the last Plantagenet paced before the royal war tent and looked anxiously across at the enemy. The view of the military map as shown in Figure 3.1 was spread out before the battle-tried leaders of the advance guard.

The army of rebels was encamped in disarray to the southwest of the swamp. At a suitably respectful distance from the Tudors' right flank the armies of the Stanleys awaited what was to come. Lost in thought, Richard fingered his nonexistent hump and wrinkled his brow into careworn creases.

Could he, when all was said and done, trust this race of Stanleys, this pillar of his kingdom upon whom honors and benefices had been heaped? William's treachery seemed certain. Even if his banishment had come too late, his three thousand men would hardly jeopardize Richard's situation. The case of Lord Stanley, the constable, was quite different. Whoever could depend on his support would surely win the day.

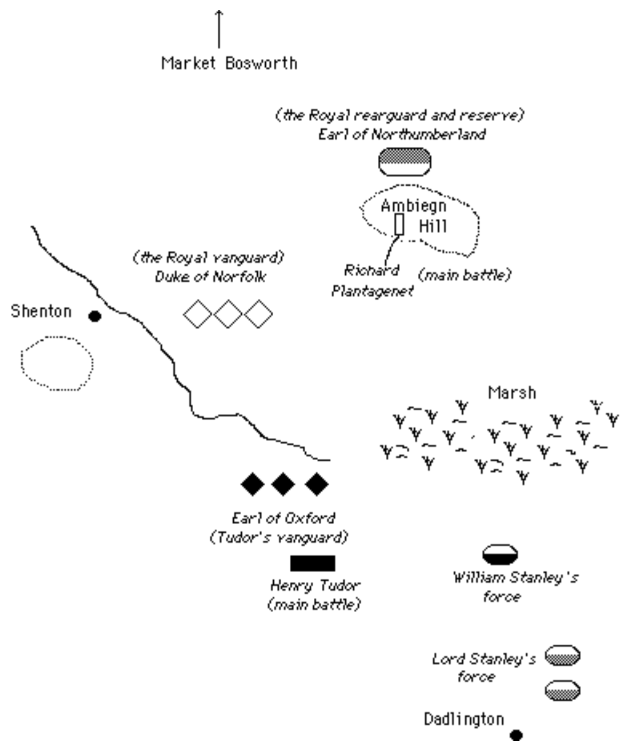

Figure 3.1. The Battle of Bosworth Field (22 August 1485)

Richard played his last trump. Before the morning had passed he sent a messenger to Lord Stanley. The message was clear and unambiguous. Should he hesitate to support his king, then Lord Strange, the king's hostage and Stanley's son, would forfeit his head.

In Figure 3.2 are given, corresponding to the three possible outcomes of the game, the valuations according to each player. Stanley clearly prefers to withhold support if he can assume that Richard will not carry out his threat. For this reason this outcome is given, from Stanley's point of view, the utility value 0. For Richard this outcome with value 0 is only the second-best outcome. He would most like to have Stanley's support; he would value this latter outcome at 5, while

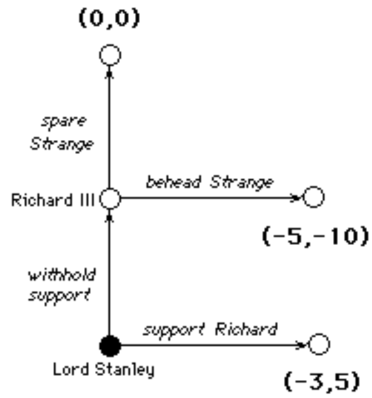

Figure 3.2. The game for Richard's last trump

Stanley gives it the value −3. Finally, both players value the execution of the hostage as the worst outcome. For Richard the utility is −10, while for Stanley it is −5.

Will the king's threat fall on fertile soil? A brief glance at the normal form representation associated to the game tree in Figure 3.2 lets us imagine the grisly outcome.

Two Nash equilibria[2] are circled in this bimatrix. In the first of these equilibria Lord Stanley gives in to Richard's threat[3] and decides to support him. The second equilibrium describes a Stanley who withholds support and a king who then does not dare to carry out his threat.

How are these two equilibrium solutions to be evaluated? The first of the equilibria is maintained only by an empty threat and therefore should be eliminated from the category of reasonable solutions.[4] A glance back at the game tree in Figure 3.2 allows us to recognize the correct way to proceed: the technique of backward induction.

[2]More precisely, the outcomes of the equilibria.

[3]In Figure 3.4 additional Nash equilibria in mixed strategies are described. Richard threatens in these equilibria to behead the hostage with probability $\frac{3}{5} \leq q < 1$ if Stanley does not support him.

[4]Together with the other threat equilibria in Figure 3.4, which are equally unbelievable.

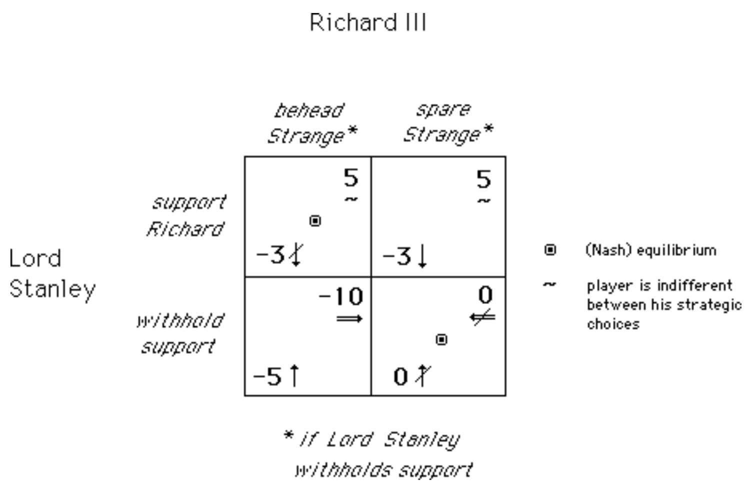

Figure 3.3. Richard's last trump—the corresponding normal form

We first consider the subgame whose root coincides with Richard's sole decision node. Confronted with the choice of whether to carry out his threat, Richard has only one remaining option: to spare Strange. Once the empty threat has been eliminated from the subgame tree on the grounds of its being a strictly dominated action, then Lord Stanley will withhold his support in the root of the original game. The resulting equilibrium (*withhold support, spare Strange*) is the only one that fulfills the property of subgame perfection.[5]

A subgame perfect equilibrium exists in every finite game tree with perfect information. For the case that no player is indifferent with regard to two different outcomes, then even the uniqueness of the subgame perfect equilibrium can be demonstrated. In the associated

[5]A subgame perfect equilibrium recommends only such plans of action that form an equilibrium in an arbitrary subgame of the original game (even in those that remain unreached in the corresponding course of the game). We have Selten [**84**] to thank for this fundamental refinement of the Nash equilibrium.

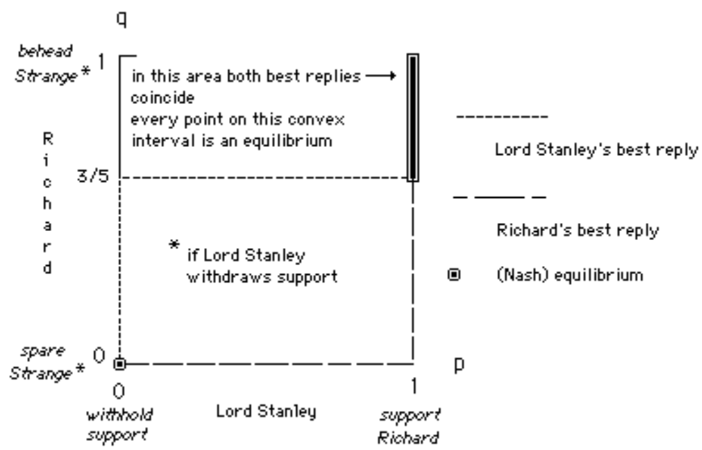

Figure 3.4. Richard's last trump—Nash equilibria

(reduced) normal form such an equilibrium will on no account contain weakly dominated strategies.[6]

Stanley's reply to Richard was short and contemptible: "I have other sons."

We assume that the bearer of this bad news returned with mixed feelings. We would like now for a brief moment to offer a different, game-theoretically motivated, turn to the actual events. On his daring ride across Redmore Plain[7] the messenger, together with his message, was overtaken by Breton marauders. This constructed incident has the most interesting consequences for the "game for Richard's last trump."

Richard has not observed his opponent's first move. His information set now consists of the two decision nodes that are connected by a dashed line in Figure 3.5. Such a game tree describes an extensive game with imperfect information.

[6]In Figure 3.3 this would be *Execute Strange if Stanley withholds support.*
[7]It was by this name that the battlefield near Bosworth was originally known.

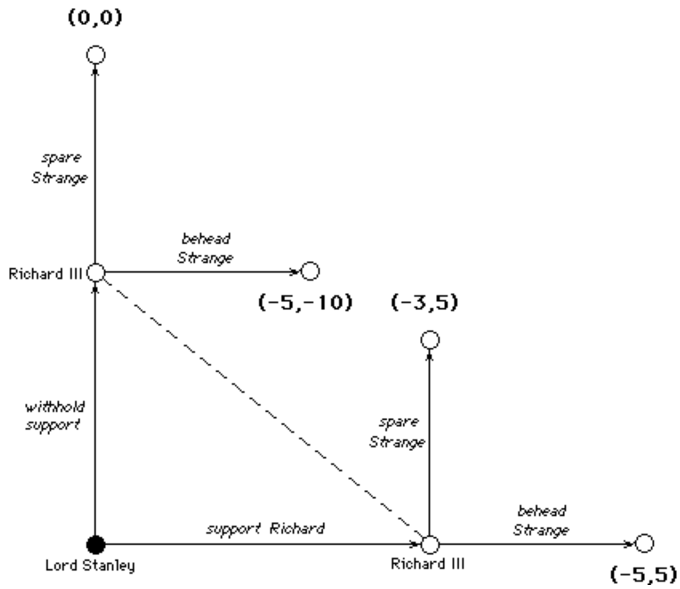

Figure 3.5. Richard's last trump—the messenger's misfortune

All the nodes that belong to the same information set of a player must lead to the same number and type of continuations.[8] The outcomes, however, that arise from the application of identical strategies at different nodes of an information set can be valued completely differently. Thus in Figure 3.5 the beheading of Strange works to Richard's disadvantage only in the case that Stanley withholds support (and Richard knows of this).[9]

In Figure 3.6 the normal form of a game with imperfect information shows two familiar equilibria.[10] The first equilibrium now consists entirely of weakly dominated strategies. We shall scarcely be able to eliminate it by means of backward induction. Namely, the

[8]In our example these are the actions *spare Strange* and *behead Strange*.

[9]Only in this case does Stanley have the option of throwing in his lot with the Tudors. If, on the other hand, Stanley has decided to support Richard, then (at least we assume so) a change of sides is out of the question. For this reason Richard would be indifferent as to his options (Figure 3.6) if he could be sure of Stanley's support.

[10]And no others, which indicates a nongeneric case.

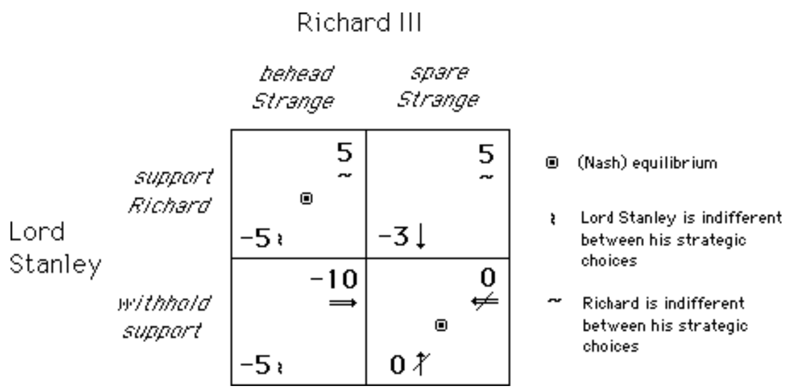

Figure 3.6. The messenger's misfortune—normal form

game tree in Figure 3.5 possesses no subgame tree other than itself.[11] This means that both equilibria are subgame perfect.

The only way that offers itself out of this dreadful state consists in a further refinement of the characteristic of the equilibrium. From among the possibilities open to us we shall for the time being bring the historically oldest into play.

In [**85**] Selten investigates the question of the robustness of an equilibrium with respect to possible errors that the players can make in choosing their actions. It is not here a question of errors in thought; we are thinking rather of a player who with trembling hand presses the wrong button on the elevator and ends up on the wrong floor.

Every equilibrium that possesses this robustness property[12] must consist of best replies to defective action plans that—if it is possible to subdue the trembling step by step until it entirely disappears—for their part converge to the strategic components of the equilibrium.

[11]Note that a decision node can be a root of its own subtree only when the information set of the player whose turn it is contains no other node.

[12]Selten calls this *perfection*, or often, to distinguish it from subgame perfection, *trembling hand perfection*

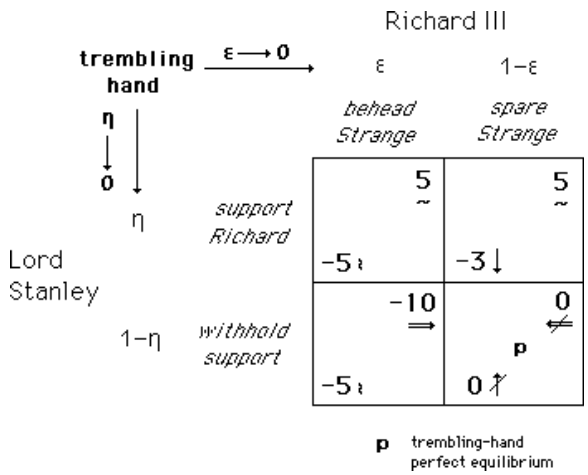

Figure 3.7. A (trembling hand) perfect equilibrium

Even if Richard is now prepared to spare Strange if need be, Stanley, on the other hand, is firmly decided to withhold his support. We know now that each of these two strategies is the best reply to the other. But what happens if one of the opponents develops a tremor?

Stanley's light tremor leads his troops with the low probability η over to Richard's side. The best reply to this completely mixed strategy is nonetheless the same for every $\eta < 1$: *Spare Strange.* On the other hand, if Richard's royal hand trembles, then Stanley's head will roll off his shoulders with the small probability ϵ. But none of this can shake Stanley's resolve. He stands, with probability $\epsilon < 1$, by his determination to withhold assistance.

Moreover, at least one sequence of pairs of fully mixed tremor strategies converges to the pair of these best replies if the trembling completely disappears.[13]

In Figure 3.7 a unique (trembling hand) perfect Nash equilibrium can be identified. What has happened to the other Nash equilibrium

[13]Choose, for example, $\eta = 2\epsilon$ to guarantee this and then let ϵ approach 0.

in Figure 3.6? In accordance with the refinement rules that we employed it must be excluded. In normal form games contested by two persons with finitely many choices of action a Nash equilibrium is (trembling hand) perfect if and only if it contains no weakly dominated strategies.

Chapter 4

Games Against Time

He thought he saw a Chapter on
   A differential game:
He looked again, and found it was
   A Long Prevailing Shame.
"A lot of reference," he said,
   "But what's about my name?"
—**Alexander Mehlmann,** *The Mad Reviewers Song*

At the border between game theory and classical applied mathematics there arose in the 1950s the theory of differential games, initially completely the work of a single individual. The concepts and ideas that Rufus Isaacs [**44**] made use of found (often under the names of others) their expression in the field of optimal control theory that was developing in parallel.

For those in the main currents of game theory the role of this theory seemed to be that of a complex and obscure collection of special cases. This accusation was partially directed at the fight and pursuit situations that stood at the center of these investigations.

In contrast to the conflict situations presented thus far, differential games stress the role of time. However, before we pursue this important influence with the help of two literary conflicts, we shall—with the help of the belligerent theory of duels—say farewell[1] to the classical zero-sum games.

4.1. Duels and Other Affairs of Honor

> Am nächsten Tag steht man befrackt in Tann
> Freds Kraftblick lässt des Gegners Schuss versagen.
> Er selbst trifft ihn am Halse überm Kragen.
> (Ein Kindermädchen trauert in Lausanne.)[2]
> —**Ludwig Rubiner et al.,** *The Duel*

In the Western *Unforgiven* Clint Eastwood plays the gunman William Munny, who at the film's dramatic climax kills five men who have drawn their guns at him. When the dust settles, Beauchamp, a witness to the dispute, asks, "Who'd you kill first? When confronted by superior numbers, an experienced gunfighter will always fire on the best shot first." To this Munny replies drily, "I was lucky in the order. But I've always been lucky when it comes to killin' folks."

Later, in connection with the three-person game truel, we shall advocate Beauchamp's point of view. In a two-person game, or duel, the question of order is beside the point, since it is always clear at whom one is supposed to shoot. Instead of order, here it is the issue of timing that hangs in the balance. Thus the duel belongs to the category of so-called timing games.

In accordance with the traditional rules of the (mathematical) two-person model, the opponents approach each other from an initial distance of A paces. We denote the first duelist's probability of hitting his opponent by $p(x)$ and that of the second by $q(x)$, where the distance between the two adversaries has decreased to x. Both probability functions increase as x approaches zero, at which point both become complete certainties.[3]

[1] These mathematical dinosaurs have already completely disappeared from the habitat of game theory and pursue their mischief only in textbooks on linear programming.

[2] They stand among the pines at break of day./ Fred's fell glance thwarts his adversary's bullet./ Then he takes aim and shoots him through the gullet./ (A chambermaid is mourning in Calais.) [Translation by David Kramer.]

[3] Since these functions are strictly monotonic, $x > y$ implies $p(x) < p(y)$ and $q(x) < q(y)$.

If we place a value of $+1$ on sole survivorship,[4] -1 on a sole close encounter with the Angel of Death, and 0 for the other eventualities, namely, that both parties survive or both perish, and if we further assume that each player has but a single bullet at his or her disposal, which when fired issues a loud report,[5] then we can calculate $N_1(x, y)$, the utility that accrues to the first duelist if x is the distance from which the first duelist fires and y the corresponding distance for his opponent, as follows.

For $x > y$ the first duelist will survive the second only if his shot (with probability $p(x)$) is a hit. If he misses (which he does with probability $1 - p(x)$), then without fear of reprisal his opponent can reduce the distance y for his shot to 0 and shoot with probability $q(0) = 1$ of success. Conversely, if $y > x$, then the first duelist will survive his opponent with probability $1 - q(y)$. If the duelists shoot simultaneously, then the first will survive his opponent only if his shot hits the mark without the opponent's shot hitting him. We therefore have

$$N_1(x, y) = \begin{cases} 2p(x) - 1 & \text{if } x > y, \\ p(x) - q(x) & \text{if } x = y, \\ 1 - 2q(y) & \text{if } y > x. \end{cases}$$

In the zero-sum game *duel* the second duelist will always strive to minimize this utility function. To this end he fires at the distance $\hat{y}(x)$ for which

$$N_1\left(x, \hat{y}(x)\right) = \min_{0 \le y \le A} N_1(x, y).$$

Now, the accuracy of the first duelist increases the longer he waits (and thus the closer he gets). Thus in no case should the distance $\hat{y}(x)$ exceed x. Let d^* denote the unique distance for which $p(d^*) + q(d^*) = 1$. Then the recipe for minimizing the utility $N_1(x, y)$ is[6]

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[4] In the interest of maintaining our PG-13 rating we shall make every attempt to depict in the sequel less bloodthirsty scenarios. In Shubik [**88**] the players are content to throw darts at balloons in place of human opponents.

[5] A so-called noisy duel, which has a considerably simpler method of solution than the other variants celebrated in Dresher [**23**] or Karlin [**47**].

[6] This corresponds more closely to a showdown on the streets of Tombstone than to a classical duel. If the minimizing duelist can tell (perhaps by a flicker in his opponent's eye) that his adversary is planning to fire from distance x, then he will also attempt to fire (and fire first) at distance x, provided that the distance d^* has been reached or passed.

$$\hat{y}(x) = \begin{cases} x & \text{if } x \le d^*, \\ 0 & \text{if } x > d^*. \end{cases}$$

The first duelist must content himself with a utility at distance x of

$$N_1(x, \hat{y}(x)) = \begin{cases} 1 - 2q(x) & \text{if } x \le d^*, \\ 2p(x) - 1 & \text{if } x \ge d^*. \end{cases}$$

Nevertheless, with a suitable choice of shooting distance he can maximize this value.

The *maximin* value of the utility function $N_1(x, y)$ is given by

$$\max_{0 \le x \le A} \min_{0 \le y \le A} N_1(x, y) = N_1(d^*, d^*) = p(d^*) - q(d^*).$$

The strategy pair $(x = d^*,\ y = d^*)$ is thereby the unique *saddle point* of the duel game, since it satisfies the following *saddle point property*:[7]

$$\max_{0 \le x \le A} \min_{0 \le y \le A} N_1(x, y) = \min_{0 \le y \le A} \max_{0 \le y \le A} N_1(x, y) = N_1(d^*, d^*).$$

In this case the *maximin* is equal to the *minimax*.[8] Therefore, the value $p(d^*) - q(d^*)$, which represents both a win for the first duelist and a loss for the second, is called the *value of the game*.

Thus does the exotic flower of paradox eke out a poor existence in the barren soil of zero-sum theory. However, the introduction of just one more gunslinger into the game suffices to alter the situation profoundly.

In a *truel* there are now three opponents facing off, each equipped with an infinite supply of ammunition.[9] Each "truelist" attempts to survive the three-way encounter.[10] We shall give our truelists the names of the three greatest actors in Western films: *John Wayne*,

[7] In a zero-sum game every Nash equilibrium has the saddle point property. Conversely, every saddle point is a Nash equilibrium.

[8] With the agreement between these two values John von Neumann has also demonstrated—in his famous minimax theorem—the existence of a game value for every finite two-person zero-sum game.

[9] In the literature (see Kilgour [**48**]) this special case is called an infinite truel.

[10] If each participant hopes to be the only survivor of the contest, then one speaks of an unambiguously antagonistic truel. If there is at least one truelist who does not care whether he alone survives or whether others survive with him, then the truel has cooperative moves.

Clint Eastwood, and *Randolph Scott*. John, let us suppose, is the best shot, followed by Clint, and then Randolph. Our truelists will stand at the vertices of an equilateral triangle, and these positions will remain fixed during the entire exchange of gunfire. Thus the probability functions for the three may be reduced to constants $j > c > r$.

The truelists begin by drawing lots for the order of firing,[11] and this order will be strictly maintained for the duration of the truel.[12]

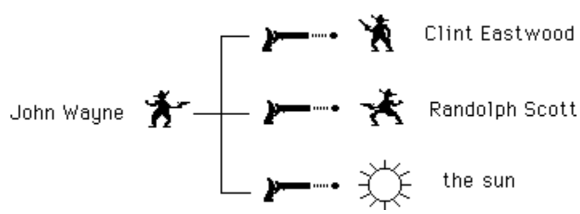

Figure 4.1. John Wayne's truel strategy

In Figure 4.1 we have shown the possible strategies for John Wayne. When it is John's turn to shoot, if both his opponents are still among the living and if he has decided once and for all to shoot in such situations at the best marksman, then he will shoot at Clint and hit him with probability j.

To be sure, John would have just as good a chance of eliminating Randolph, but in looking ahead, it becomes clear that John's probability of winning the truel would diminish in this case, since as the shooting progresses John would become the target of an opponent who is a better shot.

In an unambiguously antagonistic truel in which the only permitted targets are other players, it turns out that the strategy of shooting at the strongest opponent is always the optimal one. In Figure 4.2 we have depicted the resulting (unique) Nash equilibrium.

So, is John sitting pretty? Although John is by far the best marksman, he may find himself, as far as the probability of survival

[11] There are six possible firing orders in all.

[12] In the event that one of the gunmen bites the dust, his turn will simply be omitted.

is concerned, trailing behind Randolph. This paradoxical outcome was first described by Shubik in [**87**].

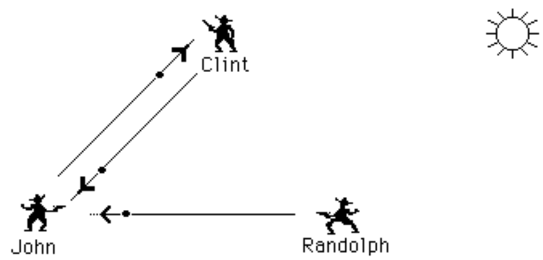

Figure 4.2. The weak are more likely to survive—Shubik's solution

For truels with cooperative moves Gardner [**31**] describes an unorthodox (additional) equilibrium in which as long as all three opponents are still alive, the third player shoots into the air instead of at one of his adversaries. The circumstances under which a voluntary waste of a shot is the optimal response to the opponents' strategies, as depicted in Figure 4.3, depend—according to Kilgours's comprehensive analysis of truels [**48**]—both on the order of shooting and on the skill of the second-best shooter.

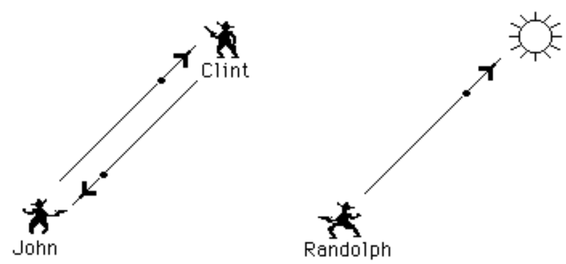

Figure 4.3. The third man—Gardner's solution

Namely, if Clint is not all that good a marksman, then Randolph, an even more pathetic shot, will always (regardless of the order of shooting) take aim at John. However, if Clint is a crack shot and has his turn directly after Randolph's, then Randolph will continually shoot at the sun until one of his stronger opponents is eliminated. At that point Randolph can take aim at the survivor and let him have it.

Donald Knuth, the creator of the magnificent typesetting language TeX (in whose refined offshoot LaTeX this book has been set), provides in [**50**] a truly pacifistic finishing touch[13] to the most cooperative of all possible truels.[14]

[13]We doubt whether the Pareto-efficient equilibrium shown in Figure 4.4 would suffice as a game-theoretic explanation for the origin of sunspots.

[14]Each player is indifferent as to whether he survives alone, or with one or even two others.

Chapter 7

Strategic Accents of Game-Theoretic Scholasticism

He thought he saw a Strategy
   Undominated, strict:
He looked again, and found it was
   Quite Easy to Depict.
"I'll never play a game," he said,
   "So simple to predict!"
—**Alexander Mehlmann,** *The Mad Reviewer's Song*

The extent to which the strategic accents of game theory become a mixed question of belief will be demonstrated in this chapter with the help of various sources of game-theoretic scholasticism.

7.1. Seeing Through the Opponent

Blackadder: It's the same plan that we used last time, and the seventeen times before that.
Melchett: E-E-Exactly! And that is what's so brilliant about it! We will catch the watchful Hun totally off guard! Doing precisely what we have done eighteen times before is exactly the last thing they'll expect us to do this time!
—**Richard Curtis and Ben Elton,** *Blackadder Goes Forth: Captain Cook*

The ability to look ahead or anticipate has already been described in all its ambiguity in Conan Doyle's *The Final Problem* [**18**].

Sherlock Holmes and Professor Moriarty confront each other in a sort of prelude and candidly discuss their common knowledge.

It is for Moriarty, the mathematician, to propose the first conjecture: "All that I have to say has already crossed your mind." To which the world's first private consulting detective answers drily, "Then possibly my answer has crossed yours."

Here intellect opposes intellect. Indeed, even the subsequent pursuit to Dover degenerates for all its dynamic drama into a masterly duel of anticipation. Holmes, whose life hangs in the balance, flees in an express train to Dover in order to reach the security of the Continent. Moriarty divines that Holmes is leaving from Victoria Station. Holmes is convinced that his foe has seen through him and will follow him to Dover in a private train.

In order to evade this move of Moriarty's, Holmes alights at Canterbury. At this point, alas, Conan Doyle alights from the carousel of anticipation in order to bring his tale to a conclusion. In order to save the good names of Moriarty and Holmes, Oskar Morgenstern, one of the fathers of game theory, continued the game of seeing through the opponent in the following manner.

If Holmes has decided to alight in Canterbury, then Moriarty should "again do what I would do" and also alight at that station. Anticipating this, Holmes should continue on to Dover, which should induce Moriarty not to get out at Canterbury, which should give Holmes the idea to disembark at that stop after all, and so on, and so on, and so on ... [1]

Thus Morgenstern arrived at the naive conclusion that there is no way out of this vicious circle of mutual anticipation. Meanwhile, however, we know better (see Section 1.1), and we ultimately have Borel [**11**] and von Neumann and Morgenstern [**71**] to thank for this knowledge. To unravel the Gordian knot of decision-making in the context of perfect anticipation we require the Alexandrian sword of chance.

Thus can the anxiety of the goalkeeper before a penalty kick ultimately be described as the fundamental fear of making the wrong

[1] (... and so on.)

decision in the cycle of anticipation.[2] The Chinese goalkeeper Gao Hong should lastly throw herself into the left corner of the goal to deflect Brandi Chastain's (United States) decisive penalty kick on the assumption that perhaps the female former forward tended to kick to the right.

On the assumption that soccer players are rational beings and moreover are inclined to plan ahead, Brandi could decide on the other corner or slam the ball down the middle of the goal. A profound analysis of further anticipation possibilities can be left at this point to much more knowledgeable sports commentators.

While we may understand the effort to see through the opponent as an important component of a strategy, the introduction of chance leads to contradictory interpretations of strategic modes of thought.

From Aumann [**2**] we have an illuminating explanation for the reasonableness of mixed strategies of normal form. The mixed strategy associated to one player will then be seen not only as a random selection from among his pure actions, but above all as a *belief* shared by all others of his patterns of behavior. Thus in a mixed Nash equilibrium every action (chosen with strictly positive probability) of a player is the best reply to his own beliefs about the behavior of the opponent.

In accord with this picture, games are above all decided in one's own head. An example that is unusual in many respects for this idiosyncratic form of game-theoretic deduction can be discovered in Gregor von Rezzori's Maghrebinian stories [**77**].[3]

One day, the rabbi of Sadagura is unexpectedly called into town. He thus finds himself confronted by the following conflict situation. The meager portion of meat that he has set aside for his midday meal will probably in his absence attract the attention of his dog, Bello.

[2] In [**25**] quoted from Peter Handke, *Die Angst des Tormanns beim Elfmeter.*

[3] The extent to which this unfortunately greatly underappreciated chronicler disguised the truth can be best judged by the author of these lines. When the Maghrebinian robber Terente (who carried out his infamous deeds not only in the Maghrebinian stories but in the real world as well) was caught in a police trap, my grandfather, at the time bearing the title of Imperial and Royal physician, was entrusted with his care. With tears pouring down his cheeks the Maghrebinian Robin Hood begged for medical treatment and underscored his request with the observation that he had been a classmate of my father's. *Non scholae, sed vitae discimus.*

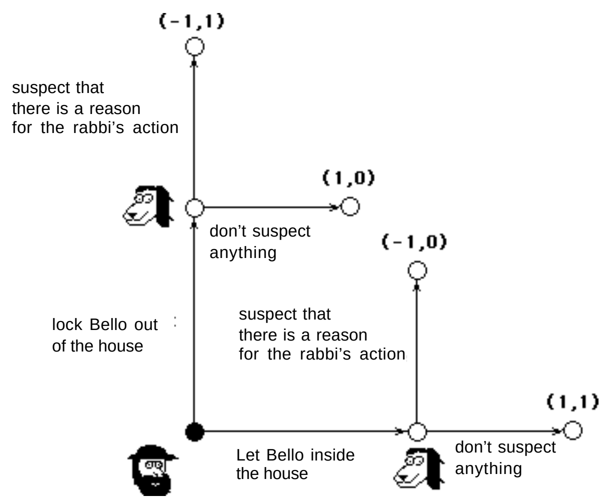

Figure 7.1. Seeing through the opponent—Maghrebinian variant

The only options that are available to him are these: (1) Lock Bello out of the house; (2) let him remain inside. If the rabbi chooses the former, then (in his opinion) Bello will at once conjecture that there is a reason for this unusual action. Therefore, he will strive to get into the house, and he will thereupon sniff out the meat and eventually devour it. If the rabbi chooses the second option, then Bello will have no reason to suspect anything and will thus neither sniff out the meat nor eat it.

In Figure 7.1 we have presented the game as it should proceed from the point of view of the wise rabbi. The solution would be to leave Bello in the house, which, so I have heard, is what indeed happened. When the rabbi returned home, Bello had eaten the meat. Then the rabbi turned to his dog, tapped him on the forehead, and said reproachfully, but gently, “Bello, you have your head screwed on backwards.”