Preface

Some years ago I started gathering nice applications of linear algebra, and here is the resulting collection. The applications belong mostly to the main fields of my mathematical interests—combinatorics, geometry, and computer science. Most of them are mathematical, in proving theorems, and some include clever ways of computing things, i.e., algorithms. The appearance of linear-algebraic methods is often unexpected.

At some point I started to call the items in the collection "miniatures". Then I decided that in order to qualify for a miniature, a complete exposition of a result, with background and everything, should not exceed four typeset pages (A4 format). This rule is absolutely arbitrary, as rules often are, but it has some rational core—namely, this extent can usually be covered conveniently in a 90-minute lecture, the standard length at the universities where I happened to teach. Then, of course, there are some exceptions to the rule, such as six-page miniatures that I just couldn't bring myself to omit.

The collection could obviously be extended indefinitely, but I thought thirty-three was a nice enough number and a good point to stop.

The exposition is intended mainly for lecturers (I've taught almost all of the pieces on various occasions) and also for students interested in nice mathematical ideas even when they require some thinking. The material is hopefully class-ready, where all details left to the reader should indeed be devil-free.

I assume a background in basic linear algebra, a bit of familiarity with polynomials, and some graph-theoretical and geometric terminology. The sections have varying levels of difficulty, and generally I have ordered them from what I personally regard as the most accessible to the more demanding.

I wanted each section to be essentially self-contained. With a good undergraduate background you can as well start reading at Section 24. This is kind of opposite to a typical mathematical textbook, where material is developed gradually, and if one wants to make sense of something on page 123, one usually has to understand the previous 122 pages, or with luck, some suitable 38 pages.

Of course, the anti-textbook structure leads to some boring repetitions and, perhaps more seriously, it puts a limit on the degree of achievable sophistication. On the other hand, I believe there are advantages as well: I gave up reading several textbooks well before page 123, after I realized that between the usually short reading sessions I couldn't remember the key definitions (people with small children will know what I'm talking about).

After several sections the reader may spot certain common patterns in the presented proofs, which could be discussed at great length, but I have decided to leave out any general accounts on linear-algebraic methods.

Nothing in this text is original, and some of the examples are rather well known and appear in many publications (including, in a few cases, other books of mine). Several general reference books are listed below. I've also added references to the original sources where I could find them. However, I've kept the historical notes at a minimum, and I've put only a limited effort into tracing the origins of the ideas (apologies to authors whose work is quoted badly or not at all—please let me know about such cases).

I would also appreciate learning about mistakes and hearing suggestions of how to improve the exposition.

Further reading. An excellent textbook is

> L. Babai and P. Frankl, *Linear Algebra Methods in Combinatorics (Preliminary version 2)*, Department of Computer Science, The University of Chicago, 1992.

Unfortunately, it has never been published officially. It can be obtained, with some effort, as lecture notes of the University of Chicago. It contains several of the topics discussed here, a lot of other material in a similar spirit, and a very nice exposition of some parts of linear algebra.

Algebraic graph theory is treated, e.g., in the books

> N. Biggs, *Algebraic Graph Theory*, 2nd edition, Cambridge University Press, Cambridge, 1993

and

> C. Godsil and G. Royle, *Algebraic Graph Theory*, Springer, New York, NY, 2001.

Probabilistic algorithms in the spirit of Sections 11 and 24 are well explained in the book

> R. Motwani and P. Raghavan, *Randomized Algorithms*, Cambridge University Press, Cambridge, 1995.

Acknowledgments. For valuable comments on preliminary versions of this booklet, I would like to thank Otfried Cheong, Esther Ezra, Nati Linial, Jana Maxová, Helena Nyklová, Yoshio Okamoto, Pavel Paták, Oleg Pikhurko, and Zuzana Safernová, as well as all other people whom I may have forgotten to include in this list. Thanks also to David Wilson for permission to use his picture of a random lozenge tiling in Miniature 22, and to Jennifer Wright Sharp for careful copyediting. Finally, I'm grateful to many people at the Department of Applied Mathematics of the Charles University in Prague and at the Institute of Theoretical Computer Science of the ETH Zurich for excellent working environments.