
Contents

Preface	ix
Chapter 1. Differential Equations	1
§1.1. Classification of Differential Equations	4
§1.2. Can we write solutions explicitly?	5
§1.3. Differential Equations as Models of Reality and Unreality	7
§1.4. Named Equations	8
§1.5. When are two equations equivalent?	9
§1.6. Evolution in Time	12
Problems	18
Suggested Reading	22
Chapter 2. Developing PDE Intuition	23
§2.1. The Structure of Linear Equations	23
§2.2. Examples of Linear Equations	30
§2.3. Examples of Nonlinear Equations	35
Problems	41
Suggested Reading	43
Chapter 3. The Story of Solitons	45
§3.1. The Observation	45
§3.2. Terminology and Backyard Study	46
§3.3. A Less-than-enthusiastic Response	47
§3.4. The Great Eastern	49
§3.5. The KdV Equation	49
§3.6. Early 20th Century	52

§3.7. Numerical Discovery of Solitons	53
§3.8. Hints of Nonlinearity	57
§3.9. Explicit Formulas for n -soliton Solutions	59
§3.10. Soliton Theory and Applications	60
§3.11. Epilogue	62
Problems	63
Suggested Reading	65
Chapter 4. Elliptic Curves and KdV Traveling Waves	67
§4.1. Algebraic Geometry	67
§4.2. Elliptic Curves and Weierstrass \wp -functions	68
§4.3. Traveling Wave Solutions to the KdV Equation	84
Problems	91
Suggested Reading	93
Chapter 5. KdV n -Solitons	95
§5.1. Pure n -soliton Solutions	95
§5.2. A Useful Trick: The τ -function	96
§5.3. Some Experiments	99
§5.4. Understanding the 2-soliton Solution	103
§5.5. General Remarks and Conclusions	109
Problems	109
Suggested Reading	111
Chapter 6. Multiplying and Factoring Differential Operators	113
§6.1. Differential Algebra	113
§6.2. Factoring Differential Operators	121
§6.3. Almost Division	124
§6.4. Application to Solving Differential Equations	125
§6.5. Producing an ODO with a Specified Kernel	127
Problems	130
Suggested Reading	132
Chapter 7. Eigenfunctions and Isospectrality	133

§7.1. Isospectral Matrices	133
§7.2. Eigenfunctions and Differential Operators	138
§7.3. Dressing for Differential Operators	140
Problems	145
Suggested Reading	147
Chapter 8. Lax Form for KdV and Other Soliton Equations	149
§8.1. KdV in Lax Form	150
§8.2. Finding Other Soliton Equations	154
§8.3. Lax Equations Involving Matrices	159
§8.4. Connection to Algebraic Geometry	164
Problems	165
Suggested Reading	171
Chapter 9. The KP Equation and Bilinear KP Equation	173
§9.1. The KP Equation	173
§9.2. The Bilinear KP Equation	181
Problems	193
Suggested Reading	195
Chapter 10. The Grassmann Cone $\Gamma_{2,4}$ and the Bilinear KP Equation	197
§10.1. Wedge Products	197
§10.2. Decomposability and the Plücker Relation	200
§10.3. The Grassmann Cone $\Gamma_{2,4}$ as a Geometric Object	203
§10.4. Bilinear KP as a Plücker Relation	204
§10.5. Geometric Objects in the Solution Spaces of Nonlinear PDEs	209
Problems	215
Suggested Reading	217
Chapter 11. Pseudo-Differential Operators and the KP Hierarchy	219
§11.1. Motivation	219
§11.2. The Algebra of Pseudo-Differential Operators	220

§11.3. Ψ DOs are Not Really Operators	224
§11.4. Application to Soliton Theory	225
Problems	232
Suggested Reading	234
Chapter 12. The Grassmann Cone $\Gamma_{k,n}$ and the Bilinear KP Hierarchy	235
§12.1. Higher Order Wedge Products	235
§12.2. The Bilinear KP Hierarchy	240
Problems	246
Suggested Reading	248
Chapter 13. Concluding Remarks	251
§13.1. Soliton Solutions and their Applications	251
§13.2. Algebraic-Geometric Structure of Soliton Equations	252
Appendix A. Mathematica Guide	257
§A.1. Basic Input	257
§A.2. Some Notation	259
§A.3. Graphics	263
§A.4. Matrices and Vectors	265
§A.5. Trouble Shooting: Common Problems and Errors	267
Appendix B. Complex Numbers	269
§B.1. Algebra with Complex Numbers	269
§B.2. Geometry with Complex Numbers	270
§B.3. Functions and Complex Numbers	272
Problems	274
Appendix C. Ideas for Independent Projects	275
References	289
Glossary of Symbols	297
Index	301