Contents

Preface vii

Chapter 1. Random Walk and Discrete Heat Equation 1

§1.1. Simple random walk 1

§1.2. Boundary value problems 14

§1.3. Heat equation 22

§1.4. Expected time to escape 29

§1.5. Space of harmonic functions 34

§1.6. Exercises 40

Chapter 2. Brownian Motion and the Heat Equation 49

§2.1. Brownian motion 49

§2.2. Harmonic functions 58

§2.3. Dirichlet problem 67

§2.4. Heat equation 73

§2.5. Bounded domain 76

§2.6. More on harmonic functions 85

§2.7. Constructing Brownian motion 88

§2.8. Exercises 92

Chapter 3. Martingales 101

§3.1. Examples 101
§3.2. Conditional expectation 108
§3.3. Definition of martingale 112
§3.4. Optional sampling theorem 113
§3.5. Martingale convergence theorem 119
§3.6. Uniform integrability 123
§3.7. Exercises 127

Chapter 4. Fractal Dimension 135
§4.1. Box dimension 135
§4.2. Cantor measure 138
§4.3. Hausdorff measure and dimension 142
§4.4. Exercises 152

Suggestions for Further Reading 155