Preface

We are drawn to the study of difference sets because this topic "belongs both to group theory and to combinatorics and … uses tools from these areas as well as from geometry, number theory, and representation theory" (quoting from the opening of Chapter 1). Each of us has supervised undergraduate research on difference sets. Our original goal in writing this book was to collect in one place the material beyond a one-semester abstract algebra course required to prepare our students for these research projects. However, the links to many parts of mathematics led to our current, broader aim: not only to serve prospective undergraduate researchers but also to provide a rich text for a senior seminar or capstone course in mathematics. With this expanded goal in mind, we highlight these mathematical interconnections.

We never intended our book to be a comprehensive survey of difference sets. However, we hope it will encourage students to explore the literature on difference sets and give them a solid foundation so they can do so successfully.

We assume student readers have taken an abstract algebra course.[1] We show them concrete examples of some algebraic ideas they studied there, and we apply and extend these concrete instances in a variety

[1] Appendix A includes the background we need from prior courses, and specific results are cited using the notation A.x.

of settings. Some of our exposition, especially in earlier chapters, is very thorough, with reasoning fully explained. The proofs of some theorems are explicitly left for the exercises, and some of these exercises offer the student considerable guidance. For other theorems we may give rather terse proofs, more like what a student would encounter in a journal article. Normally we expect the reader to fill in any omitted arguments, so we don't write "see the exercises" for each instance. In a few cases we quote theorems without proof, but always with a reference, and often with a comment on the accessibility of the proof given in the cited source.

Almost every section of the book ends with exercises. Some exercises aim to check the reader's understanding of a definition or a proof. Some ask for proofs (with or without guidance). Some are puzzles to be solved. Some invite the student to explore ideas and examples, sometimes with the aid of a computer (and so indicated). All of these kinds of exercises vary from straightforward to challenging. Appendix C includes hints for exercises marked Ⓗ and solutions to selected exercises marked Ⓢ.[2] Every chapter except the first and the last ends with a brief *Coda*[3] highlighting the main ideas and emphasizing mathematical connections.

Examples and exercises are numbered consecutively within chapters with, for example, Exercise 5 within a chapter and Exercise 7.5 for a reference to Exercise 5 in Chapter 7 made in a different chapter. Theorems are also numbered consecutively within chapters and are always referred to with both a chapter label and a theorem label, as, for example, Theorem 7.5 both within and outside of Chapter 7.

After the Introduction, Chapters 2–4 comprise the core of the book. We then see two kinds of selective paths through the rest. One would focus on representation theory and its applications. It would include Section 7.1 on intersection numbers, the constructions of difference sets in Chapters 8–9, Chapters 10–12, and Section 13.4. Another path would focus on the existence question for difference sets. It would include Chapters 5–9. Even if Chapters 10–12 are not

[2] Complete solutions are available electronically for instructors; please send email to textbooks@ams.org for more information. Some helpful computer programs are available at `http://www.ams.org/publications/authors/books/stml-67`.

[3] We borrow the term "coda" in this context from Jennifer Quinn.

covered, Sections 10.4 and 11.4 give a taste of the use of representation theory and characters in the study of difference sets. The applications in Sections 13.1–13.3 are suitable for readers following either path.

Acknowledgements.

We wish to thank the senior seminar and research students at Grinnell College and the REU students at Mount Holyoke College. Their enthusiasm inspired us, and their questions and reactions helped us shape this text. Mark Krusemeyer allowed us to borrow ideas and exercises from his Spring 2004 course on representation theory at Carleton College; we appreciate his generosity. We thank Robert McFarland for his sympathetic interest. John Polhill read several chapters and James A. Davis used parts of an early draft with an independent student; we thank them both for their encouragement. We owe a particular debt to Ken W. Smith, who read and commented on drafts of several chapters. We thank an anonymous reviewer for valuable advice on our treatment of the integral group ring. We are responsible for any errors or infelicities that remain. We are grateful for our support from the AMS: especially to Barbara Beeton for her unstinting technical assistance, to Thomas Costa for his careful and thoughtful copy-editing, and to Ina Mette for her interest and encouragement from the early days of our writing project. Finally, we thank Tom and Sandy for their love, support and many delicious dinners.

Emily Moore
Grinnell College

Harriet Pollatsek
Mount Holyoke College