
Preface

“How on earth can you do research in mathematics?”, we are often asked. The idea of ongoing research in other sciences is deeply embedded in the public consciousness, not only in those fields that may lead to self-driving cars or new life-saving drugs, but also in the most theoretical areas such as particle physics, where scientists try to puzzle out the likely nature of matter on the smallest possible scales. In contrast, even among members of the public who profess their enjoyment of mathematics there is little awareness that many mathematical questions remain open and that these are the subject of intensive investigation.

One of the difficulties in challenging this perception lies in the highly specialized nature of modern mathematics itself. Even professional mathematicians are usually unable to fully appreciate research advances outside their own branches of mathematics. Of course there are exceptions to this rule, such as **Fermat’s Last Theorem**, which was proved by Andrew Wiles in the 1990s and could hardly be simpler to state: “If $n > 2$, then there are no non-zero integers a , b , and c such that $a^n + b^n = c^n$.” Yet in this case the proof is particularly long and difficult, and only a small number of experts worldwide are able to fully comprehend it.

In the summer of 2002, the computer scientist Manindra Agrawal and his students Neeraj Kayal and Nitin Saxena achieved a remarkable feat: they discovered an efficient and deterministic test for the primality of a natural number. (To learn about the meaning of these notions, keep reading!) We were fascinated by this result not only because it answers a long-standing open question, but because the mathematics behind it is beautiful and, compared to other modern research advances, extremely accessible. We decided to use this opportunity to get young people in touch with actual mathematical research by offering a course on the subject at a German summer academy for secondary-school students in 2005. Inspired by the participants and their enthusiasm, we started to transform the course material into a book manuscript, the German version of which appeared in 2009. The book you hold in your hands is an English-language edition of this text. During the translation process, we have also corrected a few errors, changed some of the presentation for pedagogical reasons, and updated the content with new results, where appropriate.

We thank the students from our original summer course for their motivation and dedication and all those who have since helped us for their comments and suggestions regarding both the German and English editions. We also thank you – our reader – for your interest in this little volume, and we hope that you will enjoy reading the book as much as we did writing it.

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