Preface

This book introduces six selected areas of mostly 20th century mathematics. We assume that the reader has gone through the usual undergraduate courses and is used to rigorous presentation with proofs.

Mathematics is beautiful, and useful all over, but extensive. Even in computer science, one of the most mathematical fields besides mathematics itself, university curricula mostly teach only mathematics developed prior to the 20th century (with the exception of areas more directly related to computing, such as logic or discrete mathematics). This is not because of lack of modernity, but because building proper foundations takes a lot of time and there is hardly room for anything else, even when the mathematical courses occupy the maximum politically acceptable part of the curriculum.

This observation was the starting point of a project resulting in this book. Contemporary research in computer science (but in other fields as well) uses numerous mathematical tools not covered in the basic courses. We had the experience of struggling with papers containing mathematical terminology unknown to us, and we saw a number of other people having similar problems.

We decided to teach a course, mainly for Ph.D. students of theoretical computer science, introducing various mathematical areas in a concise and accessible way. With expected periodicity of three semesters, the course covered one to three areas per semester.

This book is a significantly extended version of our lecture notes. In six chapters, it deals with measure theory, high-dimensional geometry, Fourier analysis, representations of groups, multivariate polynomials (or, in fancier terms, rudimentary algebraic geometry), and topology. The chapters are independent of one another and can be studied in any order.

In the selection of the areas, the reader has to rely on our experience and opinions (although we have also asked colleagues and students what they would consider most helpful for their work). This, of course, is subjective and someone else would perhaps recommend different areas, but we believe that it is at least a reasonable suggestion—and better than no suggestion.[1]

For each of the areas, we aim at presenting basic notions, basic examples, and basic results. Exercises form an integral part of the presentation, since we believe that the only way of really grasping sophisticated notions is to actively work with them. Results too advanced to be developed rigorously in a limited space but too nice to be omitted are described, sometimes slightly informally, in encyclopedia-like passages.

Since our goal was an introductory textbook, we try to keep citations to a (reasonable) minimum. We also do not always recall notions that are usually treated on undergraduate level, since we do not want to clutter the text, and since nowadays it is easy to look up definitions in trustworthy Internet resources—which we encourage the readers to do when in doubt.

There is only so much information one can fit in a single chapter; moreover, we do not try to write as compactly as possible, preferring accessibility. Even this limited amount of knowledge can often be sufficient—as a rule, among results and notions in a given field, the simpler ones have a greater chance to be applied in other fields than the advanced and fancy ones.

[1] There is a small Internet company, whose main component is apparently an experienced man scanning and reading lots of articles from world's leading periodicals every day. For a modest yearly fee, the company then provides access to his recommendations of most interesting and important articles. We believe that, when practically any opinion can be "confirmed" by some Internet pages, such trust-based services will become more and more valuable.

Once it becomes clear that one needs to know more, one can go the "standard" way, i.e., to take a full-fledged course for mathematicians or to study a textbook. This represents a considerably greater time investment, though, and very few people have the time and energy to pursue this approach for more than two or three areas.

A possible suggestion of what to add to our list of topics is probability theory. However, we believe that probability is so crucial and widely used that it is definitely worth taking a course of one, or better, two semesters.

Prerequisites. As was mentioned above, we assume that the reader has gone through the basic mathematical courses with rigorous development, including proofs. Sometimes we use bits of mathematical analysis, discrete mathematics, and very basic probability, but by far the most important background we expect is linear algebra, including vector spaces and linear maps—indeed, in current research, one encounters linear algebra at every corner.

Readership. Our course was targeted at Ph.D. students in theoretical computer science and in discrete mathematics; the applications we present are drawn from these areas.

However, the book can be useful for a much wider audience, such as mathematicians specialized in other areas, mathematics students deciding what specialization to pursue and/or preparing for graduate school, or experts in engineering or other fields using advanced mathematics. Readers not familiar with the topics of our examples are invited to look at them, but if they find some of them incomprehensible, they are free to skip such parts.

Conventions. The main notions are set in **boldface**, less important or only tangentially mentioned ones in *italics*. Exercises are interspersed through the text, and each ends with the ✠ symbol. In the index, you will find "Fubini's theorem" both under T and under F. Mathematical symbols beginning with fixed letters, such as B^n or $\mathrm{GL}(n, \mathbb{R})$ are indexed alphabetically. Notation including special symbols, such as $X \cong Y$ or $\Bbbk[X]$ (note that we may also have $A \cong Z$ or $\mathbb{R}[Y]$, so no letters are fixed), and Greek letters are listed at the beginning of the index.

Acknowledgments. Since the chapters are self-contained, we have usually asked people to read through just one, and accordingly, most acknowledgments are postponed to the ends of the chapters.

Globally we would like to thank Tomáš Toufar for proofreading the almost-finished book.

Despite the efforts, no book is ever perfect. We would appreciate to learn about mistakes and suggestions of how to improve the exposition.

It was a great pleasure to work with people from the AMS Publishing. Especially Ina Mette was incredibly helpful, and Barbara Beeton was a model of an efficient TeX expert.

We dedicate this book to our families.